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FINAL REPORT

ON

NON-LINEAR TECHNIQUES

IN ANTENNA THEORY

Hans E. Band

Prepared By

PICKARD & BURNS, INC.
240 Highland Avenue
Needham 94, Massachusetts

Subsidiary Of

GORHAM CORPORATION

701600

Contract No. AF 19(604)-7275

September 1961

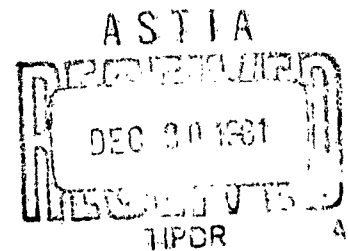
Prepared For

ELECTRONICS RESEARCH DIRECTORATE
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

P&B Pub. No. 720
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FINAL REPORT
ON
NON-LINEAR TECHNIQUES
IN ANTENNA THEORY

I. STATEMENT OF WORK

- Item 1 - Conduct a study of correlation-type antennas the end result being the experimental realization of the formulation of polynomials of signal voltages, generalizing this study to the generation of polynomials of the sums of signal voltages.**
- Item 2 - By the control of the aperture distribution, in phase and amplitude, experimentally determine optimum antenna designs for non-linear correlation-type antennas.**
- Item 3 - Develop a more universal definition of Gain and Directivity on an antenna system to include systems using correlation-type operations.**
- Item 4 - Analyze the reduction of space complexity of antenna systems by the experimental implementation of multi-frequency techniques.**

II. SUMMARY OF PROJECT

This project has essentially been a continuation of Contract AF19(604)-4535 (P&B Project P-227), entitled "Interferometer Development." It consists of four major subdivisions, as set forth in the Work Statement under Section I. All had the common purpose of experimentally and theoretically investigating the behavior of an antenna system with non-linear processing for various types of transmitted signals, for different receiving aperture distributions, and for different types of signal processing.

Three of these subdivisions consisted of theoretical and experimental work combined, where one (No. 3) was a purely theoretical and, one might also say, speculative study, awaiting future experimental confirmation or disproof. The non-linear processing used included multiplication or correlation of pairs of patterns, as well as generation of two (cubic and quintic) polynomials from the output of some simple apertures. The transmitter signals were all nominally 3000 mc, modulated in some cases with audio frequencies of 1000 cps or 2000 cps to assist in the non-linear signal processing. Transmission from a single "point" source S-Band horn mounted on an aluminum tower about 50 feet high was employed in all cases except the double-frequency experiment, and the (optional) incoherence test. For the two-frequency case, two S-Band horns mounted on the same tower radiated the frequencies 3075 mc and 3925 mc simultaneously, each modulated by the same 1000 cps audio voltage.

The over-all purpose of the investigations may be stated as follows: We have attempted to prove, for the representative cases under study, that certain non-linear operations performed on the signals derived from various simple receiving antenna elements have the effect of producing effective antenna patterns equivalent to those derived from ordinary (linearly-processed or at most square-law detected) receiving antenna systems, whose aperture length and element number exceeds that of our antenna proper by a significant ratio. We believe that the polynomial experiment provided perhaps the most striking (if imperfect) verification of this proposition. Many more experiments have suggested themselves and some of these are described under Section VI below.

III. REVIEW OF WORK ON PREVIOUS CONTRACTS, AND OF CONCURRENTLY PUBLISHED STUDIES

The preceding contract, P&B Project P-227, had as its aim the application of correlation techniques to the controlling of multi-element antenna patterns and the enhancement of the signal-to-noise ratio of signals derived from such systems. The primary objective was to produce a single-lobed, narrow beam pattern using a minimum number of antenna elements.

We carried out experiments involving the multiplying of patterns of two linear point element arrays of different but individually uniform element spacing. In other experiments we multiplied the output of a continuous aperture with the summed output of a linear, discrete, uniformly-spaced array. In yet another experiment, repeated multiplication of the output of a continuous aperture with signals from individual elements of a collinear non-uniformly spaced array was effected by a so-called "tagging method," i. e., by phase-modulating the signals of certain antenna elements at an audio rate, and later filtering the proper harmonics to extract the desired voltage product term. A multiplication technique using audio modulation (usually imposed at the transmitter) was worked out and proved to be very useful and necessary, in order to get away from having to multiply dc signals.

Theoretical work accompanying these studies included analyzing the effect of noise and interfering signals of various kinds in the performance of the systems studied. One initial attempt was made to

redefine the concepts of directivity and antenna gain for such non-linear systems, but this was mostly left for the present contract.

Another related contract with Rome Air Development Center covered the design and construction of a multiple-beam receiving antenna of approximately 3500 feet linear aperture. This work (P&B Project P-223) involved the building of two collinear, uniformly spaced arrays (the element spacings differed between arrays) similar to one of the experiments done on Project P-227 at S-Band frequency. The two arrays were respectively connected to two phase-shifting matrices whose respective pairs of outputs were then correlated in a set of 16 electronic multipliers and integrators. The effect of these operations was to produce a set of 16 non-ambiguous $1/2^\circ$ wide beams, spaced $1/2^\circ$ apart, which accepted signals over 16 individual channels at the outputs of the 16 correlators. By means of phase shifters introduced ahead of the two matrices, the set of 16 beams could be slewed up to $\pm 45^\circ$ from the array normal. The entire system was designed to operate over the range of 30-50 mc, receiving ionospheric scatter signals over a distance of about 800 miles.

In regard to outside literature in the field of the present contract, we have found that the last twelve months or so have seen a tremendous increase in articles published on the subject of non-linear processing of antenna signals in reception. It has therefore become necessary for us to restrict this survey of outside work only to the most closely related studies. A list of these papers is included in the Bibliography

in Section VIII at the end of this report. A few of these papers have already been referenced in the Final Report on Project P-227.

We would like to mention here especially the work of Welsby and Tucker¹, Welsby², Young and Ksienski³, Linder^{4, 5}, Jacobson and Talham^{6, 7}, Fakley⁸, Brown and Rowlands⁹, MacPhie¹⁰, Banta¹¹, Kock and Stone^{12, 13}, Kock¹⁴, Jacobson¹⁵, L. Davenport¹⁶, and Covington¹⁷. Space does not permit reviewing these in detail and we therefore refer the reader to the original articles for thorough study. However, we will try to summarize each article here in order to give an idea of the growth of this field of investigation. We would emphasize again that the above listing includes only the most pertinent papers which came to our attention, and is therefore not necessarily exhaustive.

Welsby and Tucker¹ in their paper "Multiplicative Receiving Arrays" studied the performance of antenna arrays in which two groups of elements are multiplied together. Directional patterns, directional discrimination, and signal/noise ratio were discussed for this type of array and often found to be mutually unreconcilable. The system is compared to superdirective arrays and a possible application to underwater echo-ranging is suggested.

Welsby² in his paper "Two-Element Aerial Array" takes up the subject of multi-frequency "carriers" used to improve the directivity of a simple array comparable to that of a multi-element additive array. A simple electronic beam-scanning system is suggested which, however, entails a lowered signal/noise ratio.

Young and Ksienski³ treat antennas as information processing devices and optimizable spatial frequency filters in their paper "Signal and Data Processing Antennas." Information theory concepts such as Information Data Rate are used in this performance optimization. The effect of noise on the optimum aperture distribution is worked out and a signal coding for maximum information content is found. Signal processing, including non-linear signal processing, is discussed and the conclusion drawn that non-linear processing degrades the information rate at low SNR's but may improve it at high SNR's. Some applications of these antenna systems are given.

Linder^{4, 5} has contributed two recent papers to the non-linear signal processing field. In "Resolution Characteristics of Correlation Arrays"⁴ he discusses arrays in which correlation techniques are used primarily to achieve very narrow beam width. The effects of two or more simultaneously present signal sources of equal frequency on the one hand, as well as of randomly varying signal voltage on the other hand, are calculated. Optimum results are found to occur for a single multiplication.

In his other paper "Applications of Correlation Techniques to Antenna Systems,"⁵ Linder treats a single point source of transmitted power, embedded in a noisy medium, which is received on a correlation type array consisting of collinear point elements oriented to place the main lobe onto the transmitter. The SNR of the combined array output is calculated as a function of individual element voltages and element

spacing, and is maximized with regard to these variables.

Jacobson and Talham^{6, 7} attack the problem of multiple receiver correlators from the acoustician's standpoint in their joint papers "Use of Pressure Gradient Receivers in a Correlator Receiving System" and "Comparison Analysis on Four Directional Receiver Correlators." (It is well to remember here that the original theoretical paper by Berman and Clay also came from the field of acoustics, but due to its general nature has found wide application.) In the first paper they discuss a correlator receiving system consisting of two directional receivers whose SNR is compared to that of a signal from a much larger number of omnidirectional receivers, also correlated. In the second paper two receivers incorporated into four different correlation systems are studied with respect to their SNR's. A single localized signal source is assumed immersed in both two and three dimensional noise fields. The directive properties of the mean outputs are then examined, and for a signal with rectangular power spectrum the main lobe widths of the mean outputs are examined as well as behaviour near the source direction. The system can be steered by mechanical rotation or by the insertion of time delays.

Fakley⁸ in his paper "Comparison Between the Performances of a Time-Averaged Product Array and an Intraclass Correlator" takes his terminology from Berman and Clay, as is evident from the title. His paper investigates three particular applications of a TAP (time averaged product) array. These are detection of a point source against

a noise background, resolution of signals from two closely spaced point sources, and an exploration of the power distribution across an extended source. Then for the sake of comparison a parallel analysis is done on an intraclass correlation system, using for simplicity an idealized four-element uniformly spaced linear array. The results are then found to have more general applicability. Repeated references are made to Berman and Clay's paper and to Melton and Karr's work on coincidence detectors (see Bibliography).

The paper "Design of Directional Arrays" by Brown and Rowlands⁹ applies information theory to the improvement of SNR of array outputs. It is found that for low SNR, the best method of increasing the information content is to add the element outputs. The improvement of directivity using non-linear operations is then taken up. For the noiseless case, patterns equivalent to an additive n-element array are shown to be synthesizable from two elements whose signals are processed non-linearly, and a maximum directivity pattern is obtained from a three-element non-linearly processed array.

MacPhie's¹⁰ report to WADC, "Synthesis of Antenna Product Patterns Obtained From a Single Array," describes product pattern synthesis achieved by cross-correlation of separate voltages derived from a single array. The voltages are formed by different linear combinations of the element voltages, and the excitation coefficients of the factor patterns are worked out and optimized with respect to minimizing the effect of a coherent interference signal simultaneously

present. A Dolph-Chebyshev synthesis is also discussed in connection with this scheme. Provision is made for shifting the radiation angle of one factor pattern relative to that of the other, mainly for interference rejection. An interesting feature we found in this paper is the inclusion of the ordinary square-law detected additive array as a "degenerate case" of the split-output multiplicative array. This provides a needed conceptual bridge between the analysis of conventional and of non-linearly processed antennas.

Banta¹¹ takes a somewhat different approach to non-linear signal processing in his paper "Far Field Properties of Wide Band Planar Arrays with Non-Linear Processing." Instead of performing the non-linear process after summation of array outputs, he reverses the order and processes each element output non-linearly before combining the resultant voltages linearly. The array is not necessarily collinear, making the treatment more general in this respect than most of the other papers discussed here. The transmitted signals have broadband character and arbitrary source spectra, which together with the receiving element distribution are found to determine the array pattern. Non-linear processing in the form of hard limiting is considered and its effect upon SNR, directional sensitivity, and angular discrimination between several sources of differing power is calculated.

Kock and Stone^{12, 13} take up the subject of space-frequency equivalence in their two papers "Space-Frequency Equivalence" and "Active Space-Frequency Correlation Systems." The first paper

discusses the correspondence between spatially complex, single-frequency antennas and spatially simple, multi-frequency antennas. The two isotropic receiver-method is generalized and extended to the two-dimensional case. A discussion of the use of multiple frequencies to obtain shaped beams and superdirective patterns from two spaced isotropic receivers is also given. The other paper is along similar lines, but brings in the active components of the system, whereby a group of frequencies spaced over several octaves is transmitted. The signals received on two or three widely spaced low directivity antennas are correlated to achieve high directionality. It is concluded that for broad transmitting and narrow receiving patterns such systems reduce antenna complexity to a significant degree.

Kock's¹⁴ paper on Sound and E-M Waves draws various analogies between the two, including polarization and cut-off effects, guided waves, etc. A section entitled "Space Frequency Equivalence" is taken from Ref. 12 (WESCON paper) above and includes a set of equivalence diagrams between a single frequency source radiating against a complex receiving array and a multiple frequency source radiating against a simple two-element correlation receiving array.

A currently (August 1961) published paper "Optimum Envelope Resolution in an Array Correlator" by Jacobson¹⁵ considers correlation of the outputs of two identical uniformly-spaced collinear arrays. The system output as a function of the individual array factors is calculated for a sinusoidal input signal. The optimization of the main lobe width-side lobe level condition is demonstrated by adjusting the complex

amplitude coefficients at each array element. The effect of this choice on the degradation of the SNR is then shown. Numerical examples are worked out for an optimum system.

The report "Data-Processing Antenna with Non-Uniformly Spaced Elements" by L. Davenport¹⁶ describes a microwave antenna receiving system which simulates a long continuous aperture. The actual antenna elements consist of a short continuous aperture and several collinear broad-beam elements (horns) of successively greater spacing. Mechanical phase shifters inserted in certain element leads and driven synchronously by an electric motor are used to "tag" these elements. After the proper correlations have been performed, the tagging frequency or one of its harmonics is filtered out to give a voltage equivalent to that of a long continuous aperture. Appropriate resultant patterns are included in the report.

Finally, Covington's¹⁷ paper "Compound Interferometer" describes the formation of single-lobed fan beam on a radio telescope antenna, using a phase-sensitive interferometer. This is the same principle (phase-shifting and synchronous detection) which Covington and Broten described earlier. The operating wave length is 10 cm and the system has been used to scan the solar disk to measure its radio noise distribution.

IV. GENERAL REVIEW OF PROGRESS ON THIS PROJECT

This section is intended as a time-saver to indicate the degree of progress which we believe we have achieved on each phase of the work. It is not meant to furnish detailed information for which we refer the reader to Section V.

Item 1. Polynomial Synthesis

This is a part of the project where we believe we have made definite progress. As seen from the antenna patterns given in the body of the following section, the simulation of a $\frac{\sin 6x}{6x}$ pattern corresponding to a 6 d aperture length has been definitely demonstrated. The polynomial variables (cosines) were voltage sums of two-element interferometers, as stipulated.

Item 2. Aperture Distribution Control

This is intimately connected with Item 4 and was in fact incorporated in it. The reason for this was that the multi-frequency technique in Item 4 at first demanded an exponential rather than constant amplitude distribution across the receiving aperture (slotted wave guide). Even though this requirement was later made unnecessary by employing two separate apertures, one of them had already been designed and constructed and we continued to use it in order to save time and funds. Thus both apertures constructed for Item 4 incorporate

a non-uniform (i. e. exponential) amplitude distribution as required by Item 4. Unfortunately time did not permit us to try other non-uniform aperture distributions for Item 2.

Item 3. Gain and Directivity Theory

The progress here has been confined mostly to examining the various more or less conflicting proposals given in the current literature as to a proper generalization of the concepts of antenna gain, directivity and efficiency, applied to non-linearly processed receiving antenna signals. In one of our Quarterly Progress Reports (No. 3) we have presented some of our own ideas on the subject, but we would wish to spend much more time on this interesting theoretical problem.

Item 4. Multi-Frequency System

In implementing the double-frequency correlation system described in detail in this report we feel that real progress has been made toward demonstrating the feasibility and basic validity of such a system in the radio wave domain. Time and funds again permitted only the exploration of one of the simplest cases, that of two frequencies, but the experience gained should enable us to expand the work to a greater number of different frequencies and other refinements such as multiple correlation, use of relative phase information, etc. in future efforts.

V. WORK PERFORMED

We have divided this part of the report according to the four items of effort described in the Work Statement. The time sequence in which the work was performed did not however correspond to the numbering of the items, but was determined by availability of personnel and equipment, relative progress made, etc. Actually no sequence was prescribed in the Work Statement, as the tasks were quite independent of one another.

Item 1. Experimental Study of Non-Linearly Processed Antenna Signals

We began the contractual study by testing some of the more general concepts of the new contract using the gear which was in use on the final phase of the previous contract (P&B Project P-227). One of these concepts (Item 4 of the Work Statement) consists of the application of multi-frequency techniques to reduce physical antenna complexity. In the initial attack on this problem we tried to realize this principle by using an auxiliary low frequency signal (called the "tagging" signal) to effect the generation of a resultant antenna pattern $\frac{\sin 16 x}{16 x}$, equivalent to the pattern of a continuous, uniformly-illuminated, aperture antenna 16 times as long as the continuous aperture used as one of our antenna elements, by operating on the signals received from seven individual antenna elements (a short continuous aperture and six S-Band horns, connected in pairs) in a certain fashion.

The details of our experimental method as well as its theoretical counterpart were given in the amended Final Report on AFCRC Contract AF19(604)-4535 (P&B Project P-227) and will not be repeated here.

We shall briefly describe here the procedure and results obtained, while referring the reader to the above mentioned report for details.

Figure 1 shows in block diagram form the sequence of operations performed. The S-Band horns had their outputs added in pairs, i. e. 1 and 2, 3 and 4, 5 and 6. After the RF signals were heterodyned down to 2 mc, a "tagging" operation was performed in two of the four

signal channels: Channel I, emanating from the continuous aperture, and Channel III, coming from summed horn pair 3 and 4. The "tagging" operation amounted to a shift of the 2 mc signal frequency to a closely adjacent frequency, here 2.002 mc. (The original 2 mc "carrier" was simultaneously present but did not affect the operation. The other "sideband" at 1.998 mc was suppressed as much as possible, since it would have led to pattern ambiguities.) Two correlations were performed, between the continuous aperture and horn pair 1-2, as well as between horn pairs 3-4 and 5-6, respectively.* The two resultant correlator output signals now had a frequency equal to the tagging frequency. They were in turn correlated in a final correlator stage where the double-tagging frequency output was phase-detected by a signal derived from the local audio oscillator supplying the tagging signal and a frequency doubler stage. The final result of all this was a resultant $\frac{\sin 16 x}{16 x}$ pattern, as is derived in the Final Report mentioned above. Also in this report is a description of the experimental procedure and the numerous difficulties we encountered. The latter seemed to be primarily due to frequency instability of the S-Band transmitter and the local oscillators used for heterodyning the received signal to 2 mc.

The "tagging" experimentation described in the foregoing paragraphs turned out to be unsuccessful for several reasons, into which we may go briefly.

* The correlators here are merely multiplier-integrators with suitably chosen time constants.

Our first method of "tagging" used a set of "Variogon" phase shifters which were rotated mechanically by an electric motor. We were operating at an IF of 2 mc, and the maximum frequency shift was of course determined by the highest admissible shaft speed of the Variogon. This had to be kept to less than 100 cps to avoid mechanical failure of the phase shifters, giving a percentage frequency shift of only 0.005%, an obviously insufficient amount. We then attempted the construction of an all-electronic phase shifter designed to operate with a frequency shift of about 1-2 kc, driven by an audio oscillator and using synchronous detection. The difficulty with this device lay mostly in the strong dependence of phase shift on carrier frequency (here the IF), whereas the IF signal had a persistent tendency to drift out of the narrow pass band of the IF amplifiers. Since similar experimentation was going on at AFCRC under Messrs. Drane and Davenport, it was mutually agreed that we should discontinue pursuing the "tagging" method of non-linear processing and turn to the other schemes which needed investigation.

Extended aperture simulation, using polynomial operations on interferometer signals, thus became our next objective. The signals came from a two-element (point-source) interferometer consisting of two S-Band open-ended wave guides spaced a distance $d = 5 \lambda$ and connected for "total power," i. e. summation. This provided the cosine dependent signal ($\cos u$) to our "Polynomial Generator."

Using the trigonometric identity

$$\frac{\sin 6\mu}{6 \sin \mu} = \frac{1}{3} (16 \cos^5 \mu - 16 \cos^3 \mu + 3 \cos \mu), \quad (1)$$

where

$$\mu = \frac{\pi d}{\lambda} \sin \theta, \quad d = \text{horn spacing for added outputs.}$$

we tried to simulate a six-fold expanded continuous aperture.

For this purpose we constructed a simple analog function generator, using commercially available packaged units in combination with suitably designed networks. The packaged units selected were K2-W Operational Amplifiers, K2-P Stabilizing Amplifiers and K2-B1 Booster Followers manufactured by G. A. Philbrick Researches. These units were fed from a Philbrick Regulated Power Supply, type R-100B. Going into more detail, we note that our polynomial generator consisted of a chain of power law function generators whose outputs were fed through appropriate coefficient units to an adding circuit which had the complete polynomial as its output.

The power law functions were generated by use of straight line segment approximations. The accuracy depended on the degree of the power law and the number of line segments used. This was accomplished by the use of standard equipment and a simple design procedure which guaranteed ease of adjustment and good reliability.

The polynomial function was generated as follows:

1. Produce the needed powers of x^n
2. Multiply the powers of x^n by their respective coefficients A_n

3. Add the terms $A_n x^n$. The output of the adding circuit yielded the complete polynomial $f(x) = \sum_n A_n x^n$.

Generation of the power law functions could be done by two means:

1. Parallel connections
2. Series connections

This is illustrated in Figure 2 below.

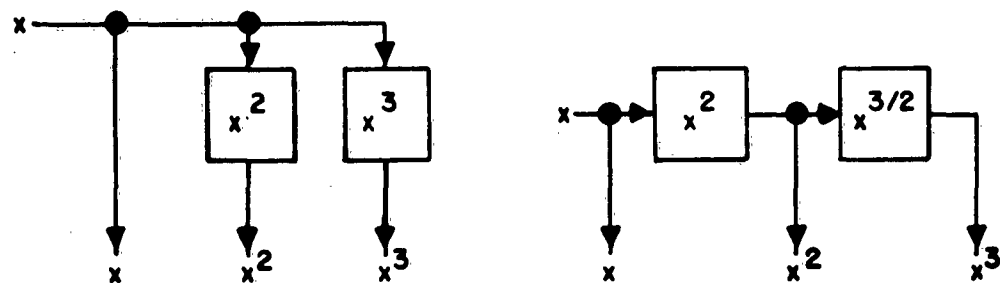


FIGURE 2. METHODS OF CONNECTION

The big disadvantage of the parallel connection was the difficulty encountered in making the higher power law generators. The difficulty was more pronounced beyond the second degree law because of the rapid change of the derivative which imposed design problems. By duality we saw that the big disadvantage of the series connection was the problem of the decreasing derivative as $x^{\frac{n}{n+1}} \rightarrow x$ for $n \rightarrow \infty$. This low value of slope (derivative) would have required large series resistors of value comparable to the back resistance of the diodes, a condition to be avoided.

The real big advantage of a series connection was the use of easily generated functions to produce more difficult ones. The degree of the polynomial was a big factor in deciding which approach should be used, and the best choice seemed to be compounding the series and parallel methods. We first generated the required powers of x . Then each power of x was combined with its coefficient A_n and added (see Figure 3).

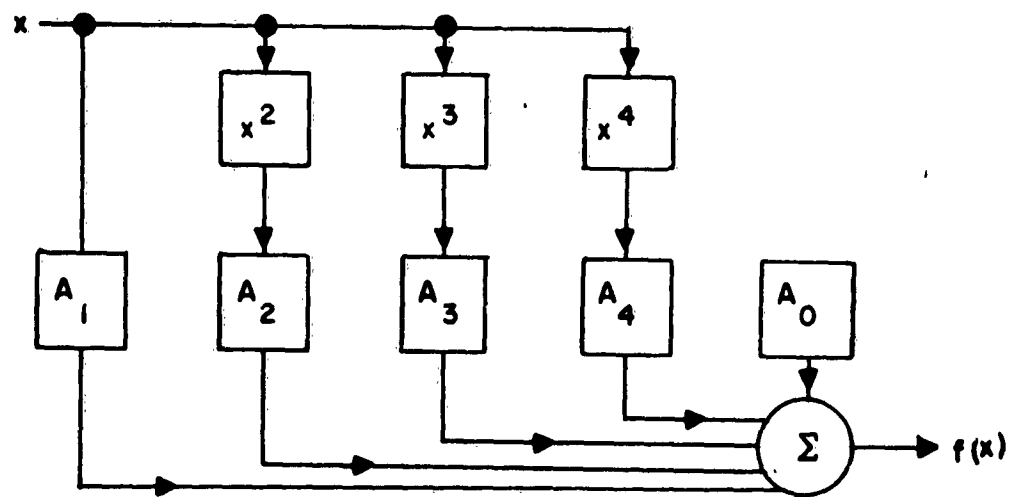


FIGURE 3. SERIES-PARALLEL POLYNOMIAL GENERATOR SCHEME

The over-all system was not complicated from the functional viewpoint, although there were practical design limits. These included voltage limits on the output and input signals. The way out of this difficulty was to scale the circuitry according to the limits of the analog system response.

We produced the non-linear power law by the use of piecewise linear approximations and used linear means the rest of the way to compose the polynomial.

For powers x^n with n greater than 1, we know that the first derivative of the curve increases with x . This fact required the power raiser to be an amplifier whose gain increased at the appropriate break-points of a set of straight line segments. The design of the generator was based on this idea.

The basic components consisted of a Philbrick Operational Amplifier, Type K2-W, which is a compact plug-in unit requiring appropriate power supplies, and an external resistor network. The amplifier unit is illustrated below. (Figure 4)

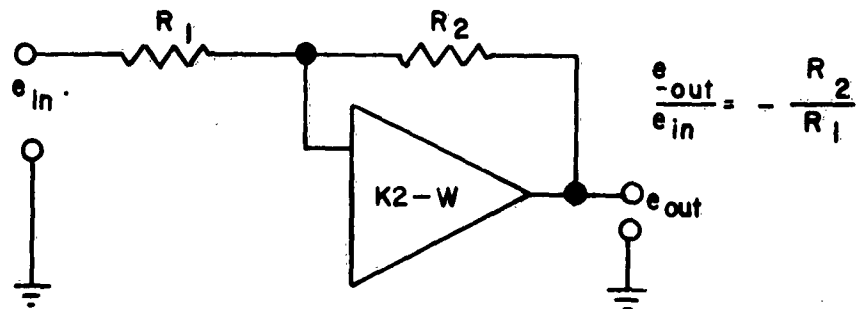


FIGURE 4. PHILBRICK AMPLIFIER SYSTEM

The amplification is 180 degrees out of phase with the magnitude of the gain set by the resistance quotient. Therefore, the net input voltage to the Philbrick unit will tend toward zero and the unit may be considered a servo unit which keeps its input voltage at zero by varying the output voltage to offset the error introduced by the input signal. Hence the resistance quotient determines the gain.

Because of this fact, we may increase the gain by either resistor, e.g., (a) enlarge the output resistor; or (b) reduce the input resistor. The first idea was difficult to realize, hence, we

decreased the input resistance by adding other resistors in parallel. Next, the addition must take place when the input voltage reaches the appropriate level. This was accomplished by the use of a diode, as shown in Figure 5.

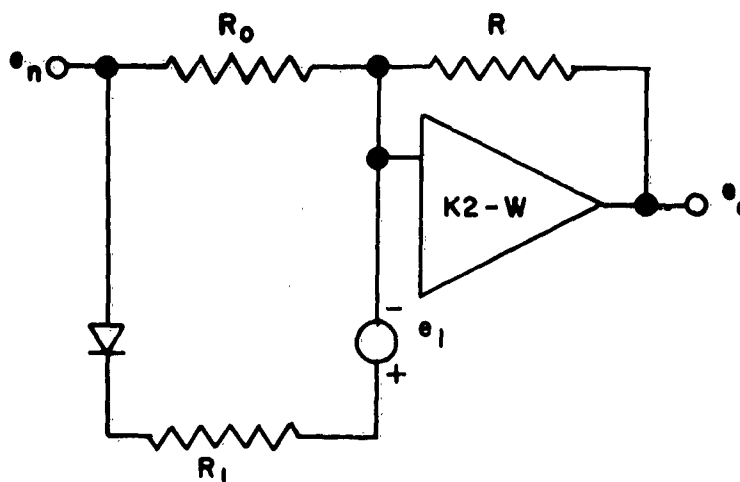


FIGURE 5. PIECEWISE LINEAR FUNCTION GENERATOR

By circuit analysis we observe the following:

$$\begin{aligned} \text{Gain} &= R/R_0 \text{ (by analogy to Figure 4)} \\ &= M_1, \text{ the slope of the first line segment} \\ &\text{when } e_{\text{input}} \leq \Delta e_1. \end{aligned}$$

Now the first break-point where the line segment slope changes is

$$e_{\text{input}} = \Delta e_1.$$

Now we have a new condition of the circuit described by

$$\begin{aligned} \text{Gain} &= R \left(\frac{1}{R_0} + \frac{1}{R_1} \right), \\ &\text{when } e_{\text{input}} \leq \Delta e_1. \end{aligned}$$

By adding more networks (i. e. diodes, resistors and voltage sources), we have the generalized description as follows:

$$\text{Gain} = -R \sum_{i=0}^{n-1} \frac{1}{R_i} = M_n$$

when $\Delta e_{n-1} \leq e_{\text{input}} \leq \Delta e_n$.

Next, we solve for the expression R_n in terms of M_n , getting the following expression for the nth resistor:

$$R_n = \frac{R_{\text{output}}}{|M_n - M_{n-1}|} \quad \text{ohms} \quad (2)$$

$$n \geq 0.$$

The expression for R_n is all that is necessary to compute the shunt resistors of the diode network.

Figure 6, Power Law Function-Segmented Approximation, is the schematic of a function generator. Note how the Δe_n voltages are derived from a floating voltage divider. It is of importance to include this resistance when calculating the shunt resistors, R_n ; hence, it is desirable to have the voltage divider resistor appreciably smaller than the diode network resistors. Except for design limits and problems encountered, this completes the general description of a simple power law function generator. The limitations of the unit may be expressed as follows:

- a) Output current ≤ 1 milli-ampere
- b) Sum of the parallel diode back conductances $\ll 1/R_0$.

These two conditions place limits on the two important design parameters (a) degree of power law and (b) degree of accuracy. The maximum deviation from the ideal power law curve was about 5%. Note that the bias

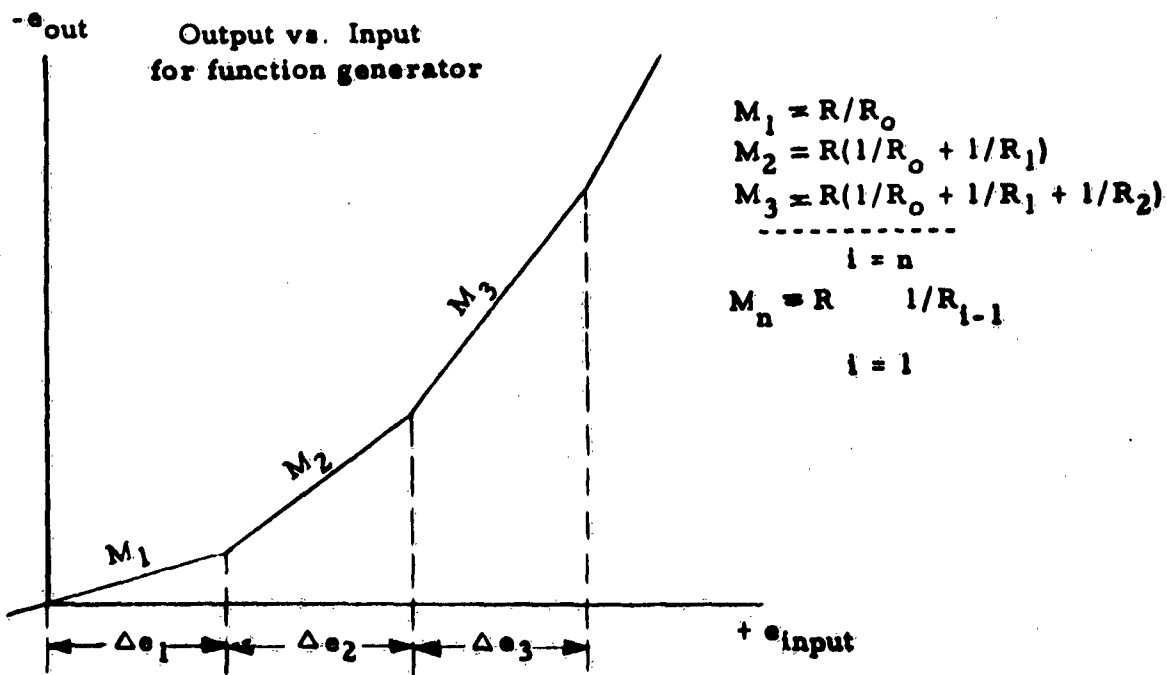
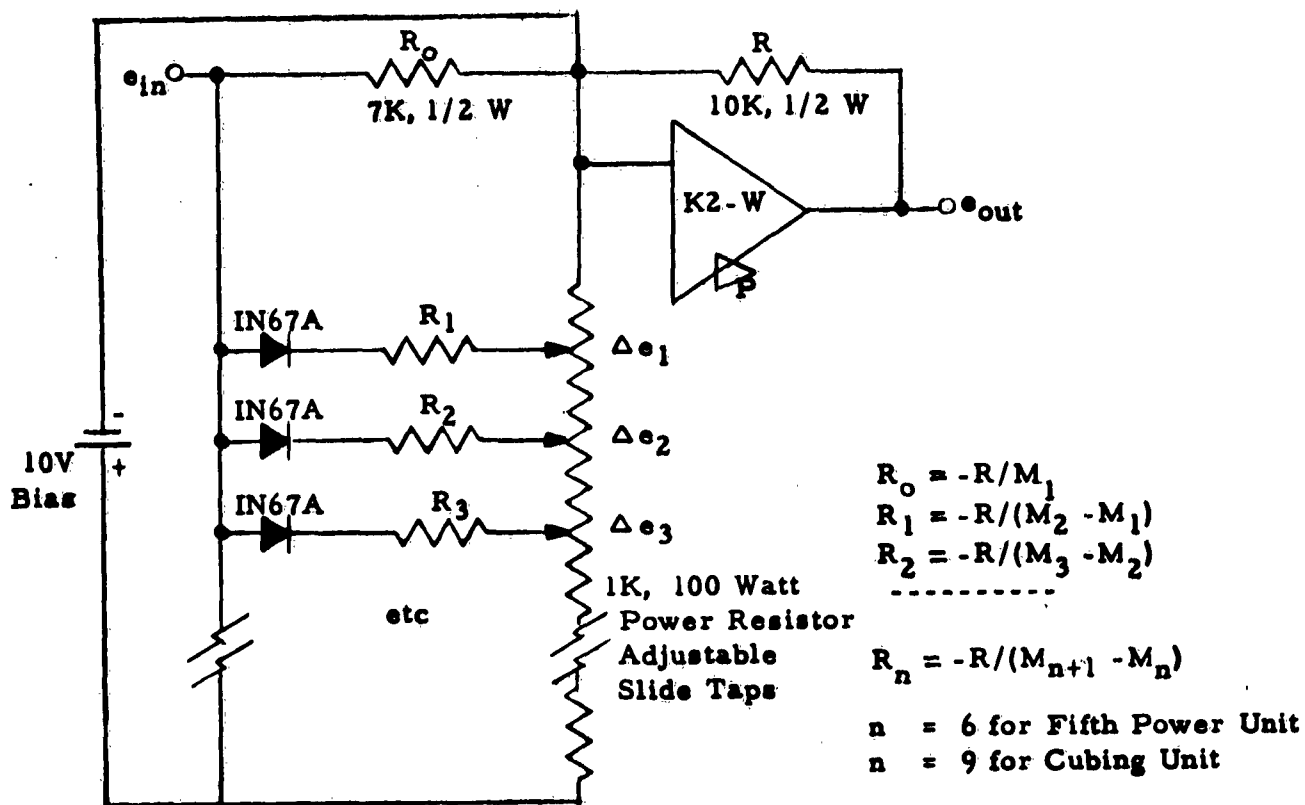


FIGURE 6. POWER LAW FUNCTION - SEGMENTED APPROXIMATION

voltages Δe_n need adjustment because the diode break-points are not quite at zero volts but at a small fraction of a volt.

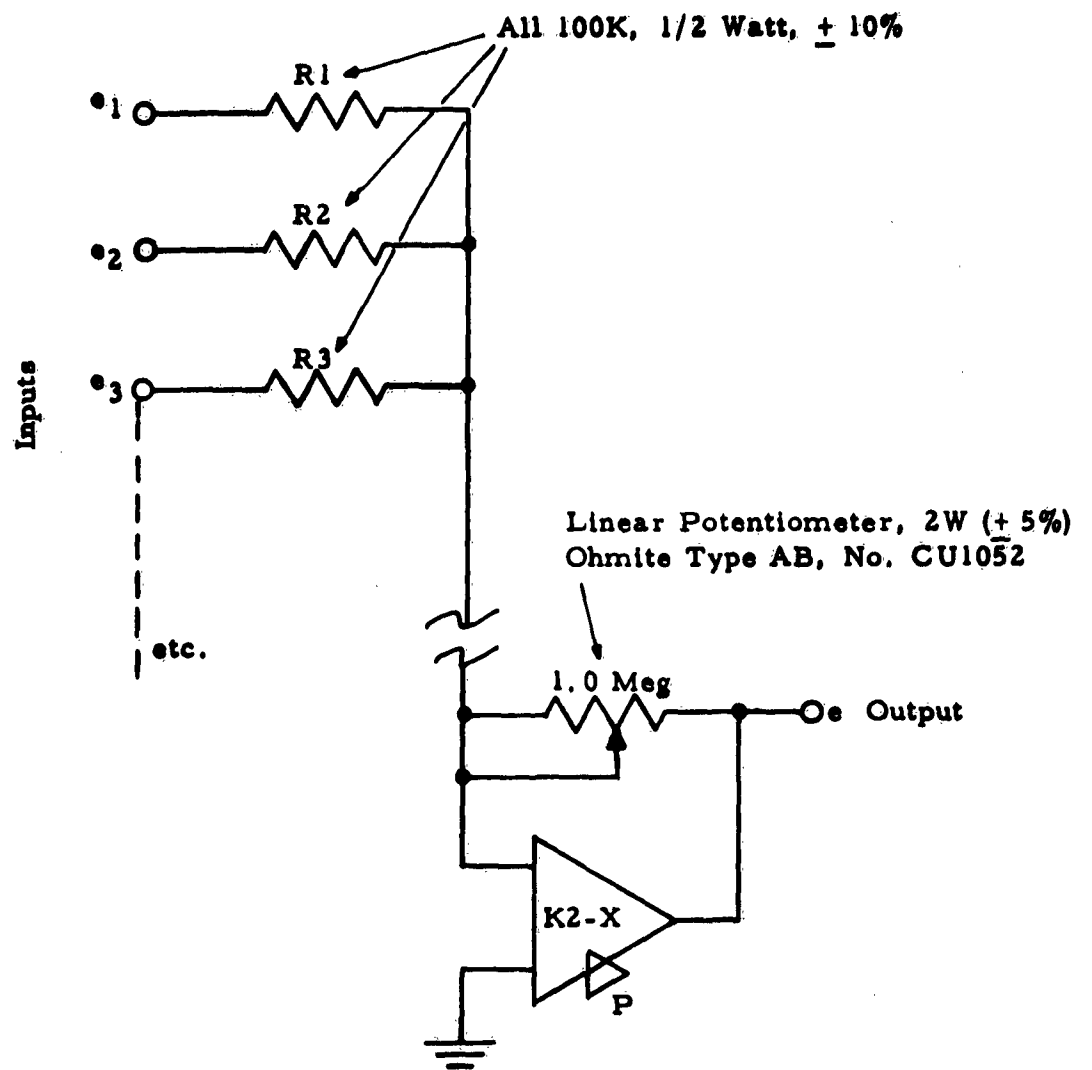
The problem of output voltage offset and drift was reduced to 1.3 millivolts or less by a Philbrick K2-P Stabilizing Amplifier unit working with the K2-W amplifier unit. Hence, the reference was established approximately at zero volts output for no input signal.

We operated the function generator with a Philbrick K2-B1 Booster-Follower getting an output current of 25 milli-amperes. The range of the first derivatives was thereby increased and consequently the higher degree power law function generators became practical.

Another alternative we tried consisted of a set of cascaded (series) function generators as described above. These were used in making a polynomial function generator for this contract. It presented an economical advantage because Booster Followers type K2-B1 need not be used at every power law unit (see Figure 2).

The linear combining unit which adds the polynomial terms is shown in schematic form in Figure 7. A K2-X Operational Amplifier (with K2-P stabilization) was used since this unit was available, but a K2-W unit would have served just as well.

Actual construction and trial proved the unit to be successful in design and ease of operation. We consulted with Mr. B. J. Vachon, of George A. Philbrick Researches, Inc. in regard to the design. By way of contrast, commercial line segment function generators, such as Philbrick's Model FF Arbitrary Function Component, are very costly. By comparison, our cost was much lower in materials and parts.



NOTE: All Resistances Rated in Ohms

FIGURE 7. LINEAR COMBINING UNIT

For the simplest case to be tried, the input to our function generator consisted of a cosine signal derived from multiplying or adding the outputs of two of the 3000 mc antenna horns employed on the previous contract.

It is worth noting that the polynomial generator was designed to operate from a fairly high impedance dc source of single polarity, the signal multiplier or correlator. It was therefore intrinsically not suitable for processing alternate polarity or ac signals. We mention these facts because Item 4 of the Work Statement calls for the implementation of multi-frequency techniques. We therefore point out that in this case we detected the individual frequency signals before any analog operation was performed on them, such as in the polynomial generator. The apparent restriction of requiring a single-polarity input to the analog device is related to the question of negative antenna pattern "lobes" resulting from, say, multiplication of out-of-phase signals. That is to say, we must decide whether the rejection of negative dc input signals, such as those due to negative lobes, constitutes a loss of needed information. Covington and Broten (PGAP Transactions July '57) have commented on the presence of negative side lobes, but no conclusions were drawn.

Figures 8 and 9 show the polynomializer's response as a function of incidence angle θ and input voltage $V = \cos \theta$. Front and back views of the complete Polynomializer are shown in Figures 10 and 11.

We may mention here the possible usefulness of being able to distinguish between (adjacent) positive and negative lobes of an antenna, e. g. to cut the azimuth position ambiguity for the detection of a

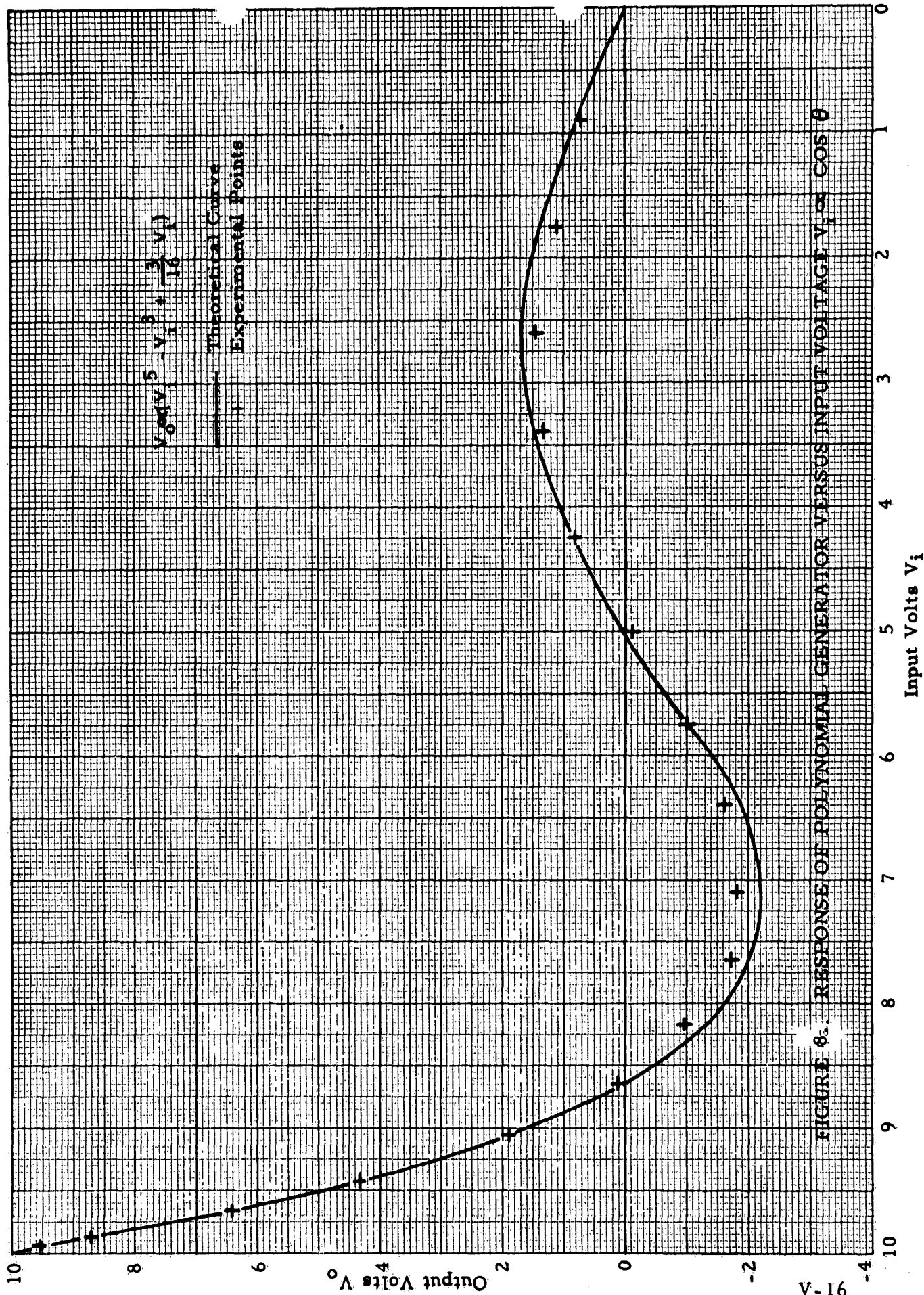


FIGURE 8. RESPONSE OF POLYNOMIAL GENERATOR VERSUS INPUT VOLTAGE V_i OR $\cos \theta$



← Input Volts V_i

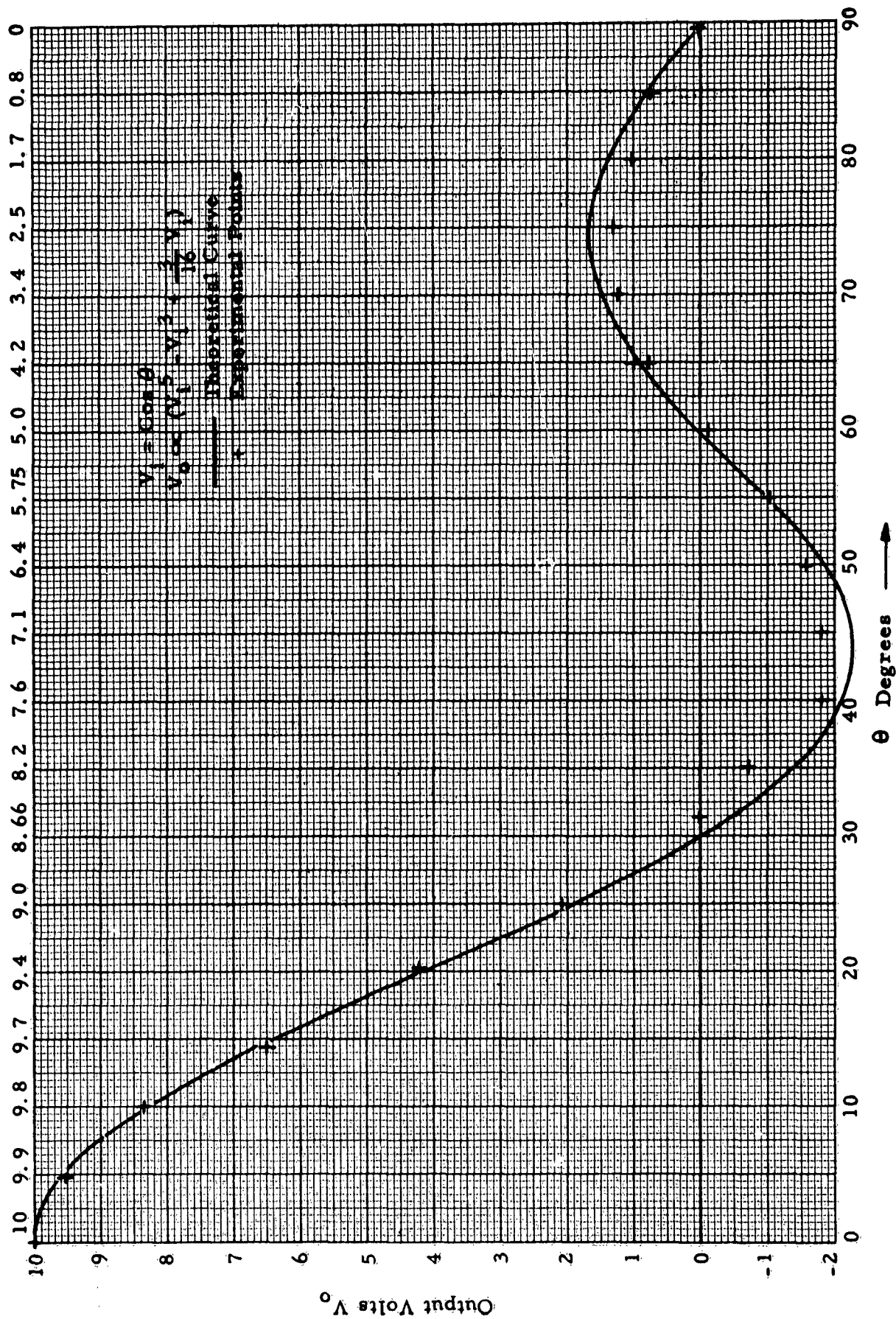


FIGURE 9 RESPONSE OF POLYNOMIAL GENERATOR VERSUS PHASE ANGLE θ

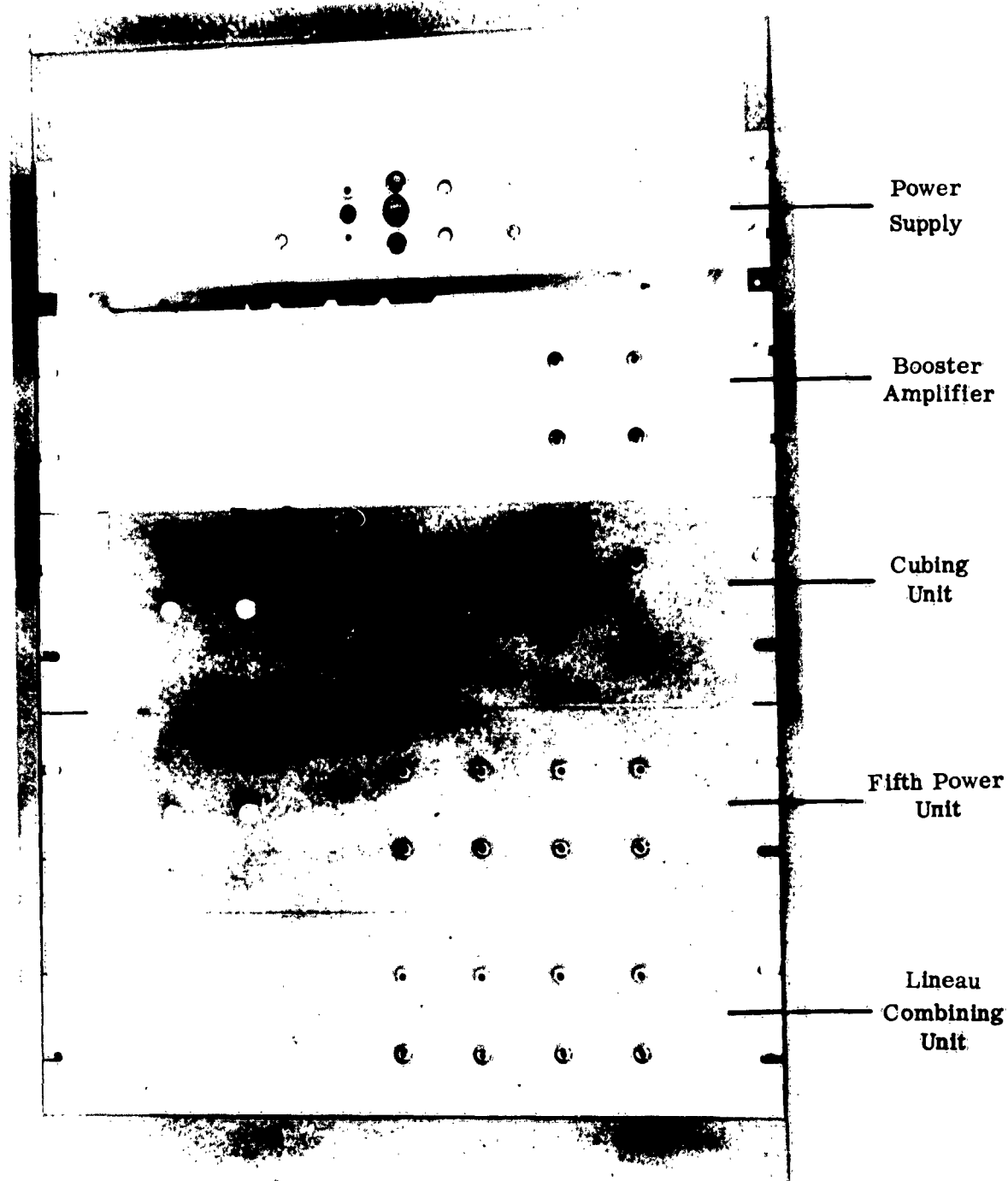


FIGURE 10. POLYNOMIALIZER (Front View)

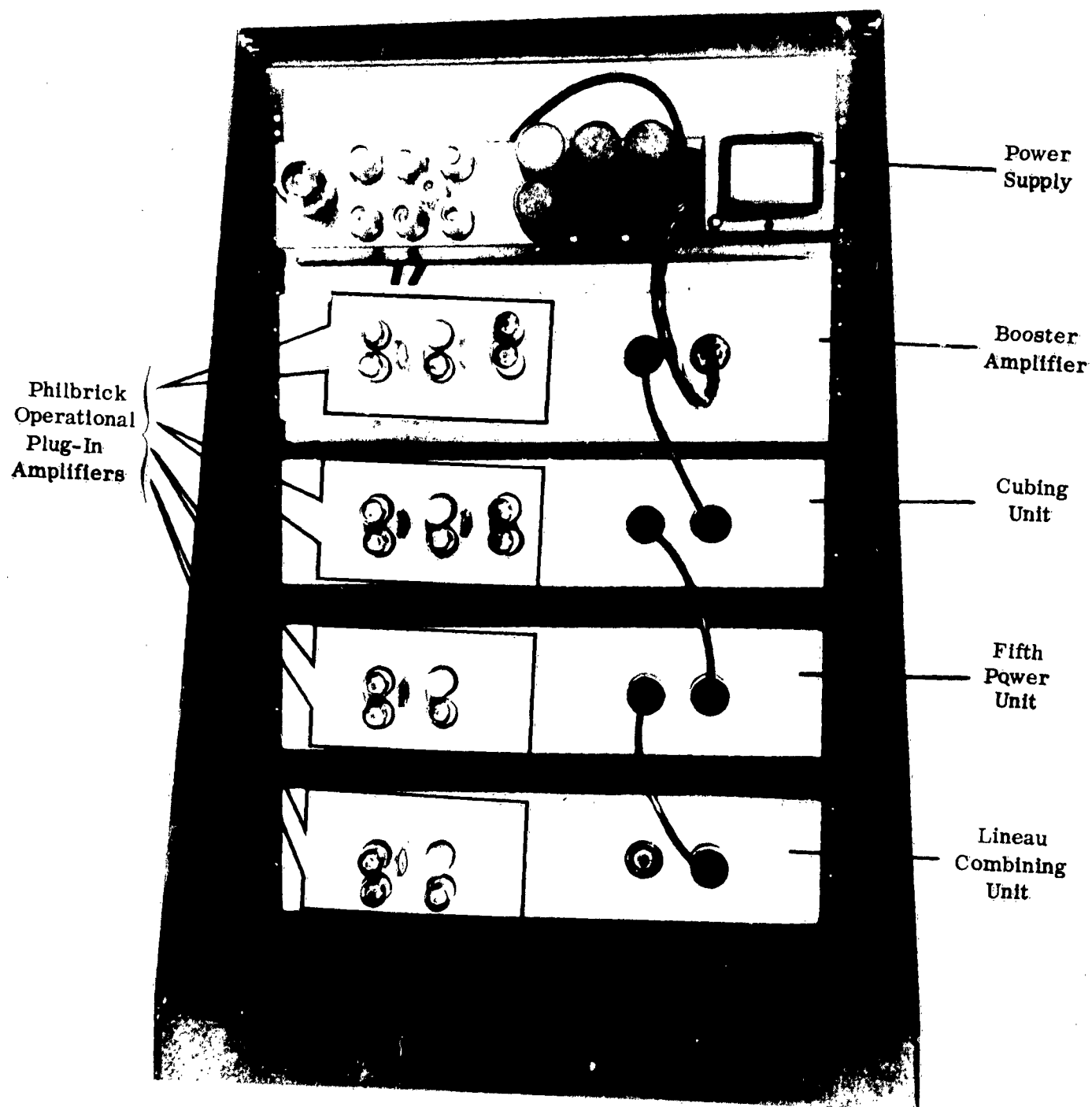


FIGURE 11. POLYNOMIALIZER (Rear View)

transmitting point source roughly in half. This could be accomplished by comparing the phase of the received signal with that of a reference signal from a local oscillator connected by equi-phase lines to all antenna elements. The sign of their phase difference would then determine the "polarity" of the lobe which is receiving the signal.

In regard to the receiving antenna arrangement used as input to the polynomial generator, we recall that two 3000 mc open-end wave guide horns can be connected either for signal multiplication or for "total power" addition. Since only two horns were used, a phase error in their lead-in lines, especially at S-Band frequency, can be very serious in throwing off the pattern direction. We therefore decided to switch to semi-rigid "Foamflex" coaxial lines for connecting the horns to the microwave receivers. Special Gremar connectors for the Foamflex cables were therefore installed and used on the first full-scale test of the polynomial circuitry working with the antenna.

We proceeded to use a two-element 3000 mc interferometer array receiving radiation from a six-foot parabolic dish mounted on a truck a distance of approximately 640 feet away. At this distance the departure from a plane-wave was calculated to be small enough to give lower errors than those from other parts of the system. We used a self-contained mobile power unit to furnish transmitter power, so that the truck could be moved around at will and trailing cables were avoided. This had become quite a crucial matter since much building activity

was then going on behind our laboratory with its attendant interference, in the form of trucks, etc., crossing near or through our transmission paths.

As we mentioned before, the basic receiving system consisted of two 3000 megacycle horns connected for addition. This was to generate a simple cosine pattern for an input to our polynomial generator. We had also planned to multiply the two horn outputs to give a cosine pattern of twice the argument as that from the additive arrangement. However, our multiplier which never worked too well at 2 mc had been redesigned to work at 30 mc, the first IF, but its operational testing was then incomplete and it could not be applied to these experiments.

One of our set-ups is shown in block form in Figure 12. After being heterodyned down to 30 mc in the two microwave receivers, the two signals were added in a tee and detected in a specially built linear detector (rectifier). The schematic of this is shown in Figure 13.

This device was necessary in order to obtain the desired $\cos x$ instead of the usual square law detected $\cos^2 x$ voltage. The detector output was then amplified in a Hewlett-Packard Model 425A DC-Amplifier.

At the output of the latter we provided two alternative connections:

1. The "polynomializer" (i. e. polynomial generator)
2. Straight feed-through to the next stage.

The next stage consisted of a transistorized square wave chopper, built especially for this purpose, and driven by an audio oscillator set

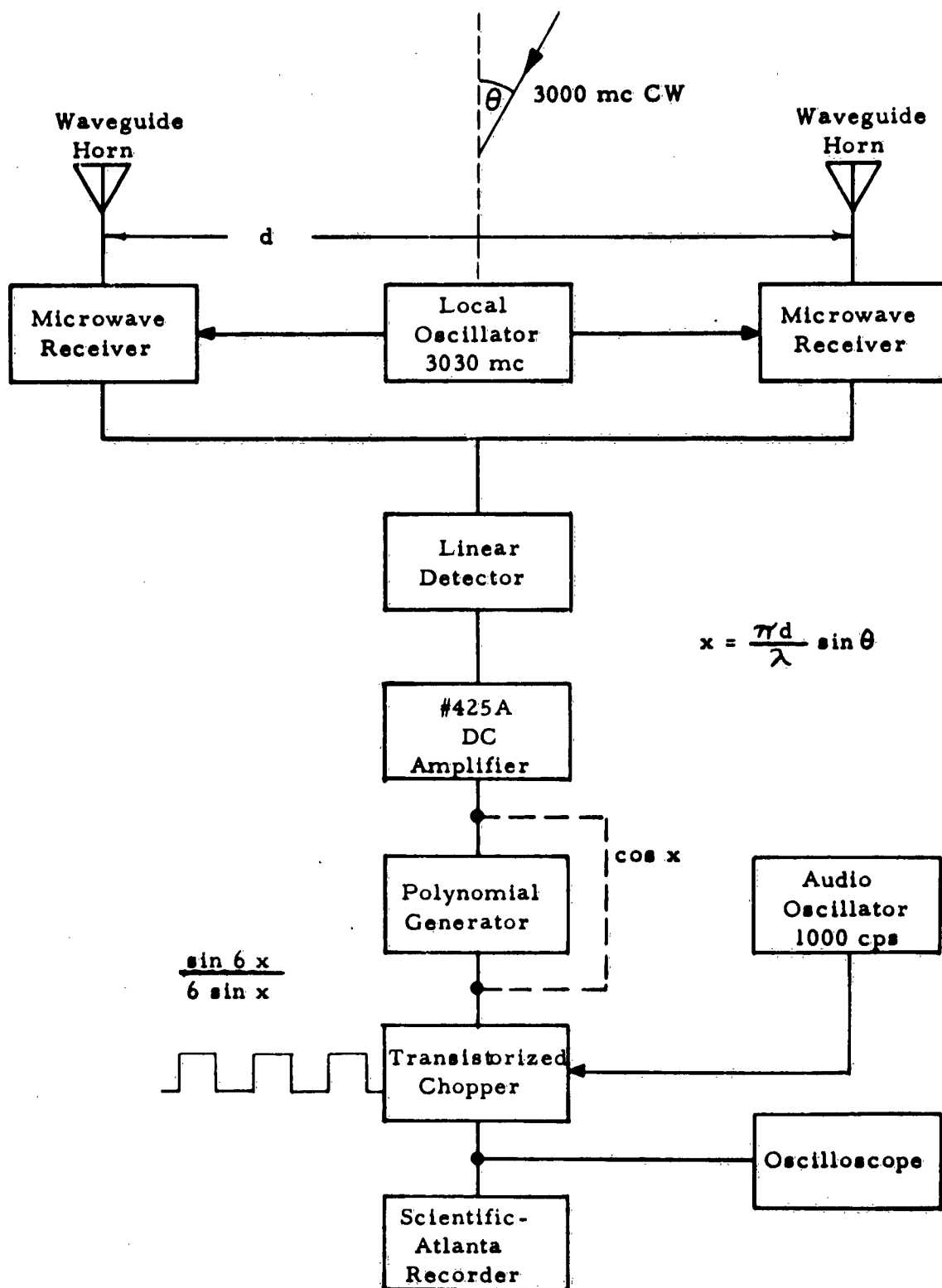


FIGURE 12. ADDITIVE INTERFEROMETER WITH POLYNOMIAL SIGNAL PROCESSING

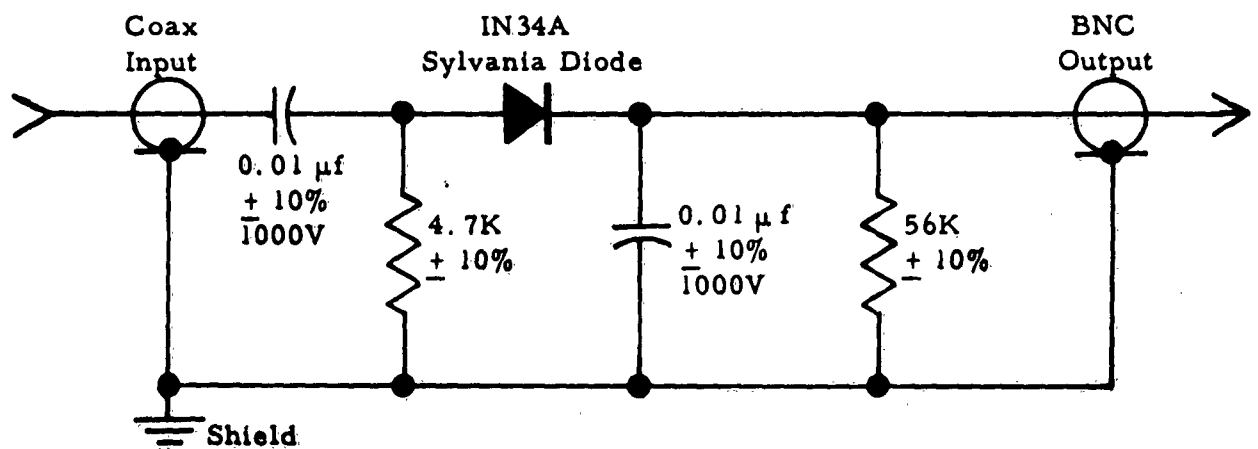


FIGURE 13. LINEAR DETECTOR

at 1000 cps. The purpose of this was to produce a 1000 cps square wave, amplitude proportional to the dc signal, which would be accepted by the Scientific-Atlanta Pattern Recorder.

Two of the patterns taken with the experimental arrangement just described are given herewith. For this particular experiment we used a horn spacing of $d = \lambda = 10$ cm. The additive pattern is shown in Figure 14 while the same signal passed through the polynomializer produced the pattern shown in Figure 15. The (voltage) side lobe level of the latter pattern reads about 4.5 db, but this must be doubled since a linear instead of a square-law detector was used to drive the pattern recorder.

Thus, an effective side lobe level of about 9 db was obtained in this particular experiment. This should be compared to the theoretical side lobe level of 12.4 db for a $\frac{\sin N x}{N \sin x}$ pattern at $N = 6$.

The location of the peaks also agrees quite closely with the theoretical values, as much as peaks are distinctly visible in the pattern of Figure 15. Due to a slight phase error in one of the signal channels, the center of the main lobe fell about 6° to the left of the chart zero, and hence it is best to measure the relative peak spacings here.

Many other patterns of equal or lower quality were obtained at other horn spacings, one going up to $d = 6 \lambda$. These were compared to theoretical curves and found to fall short of the idealized side lobe level by about the same percentage as for $d = \lambda$. Relative

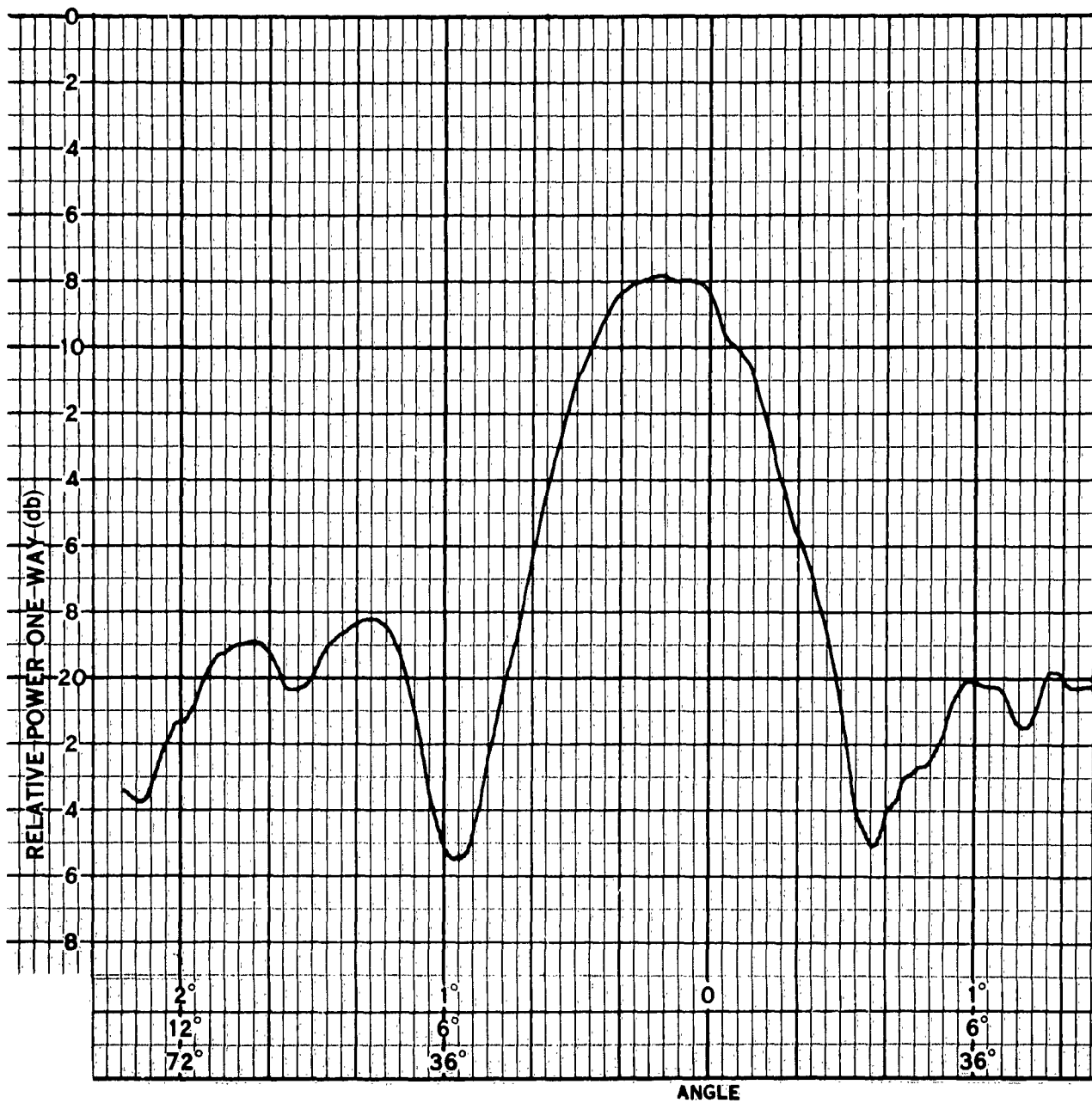


FIGURE 14. RECTIFIED VOLTAGE PATTERN FOR $d = \lambda = 10$ CM
TWO ELEMENT INTERFEROMETER (LINEAR DETECTION)

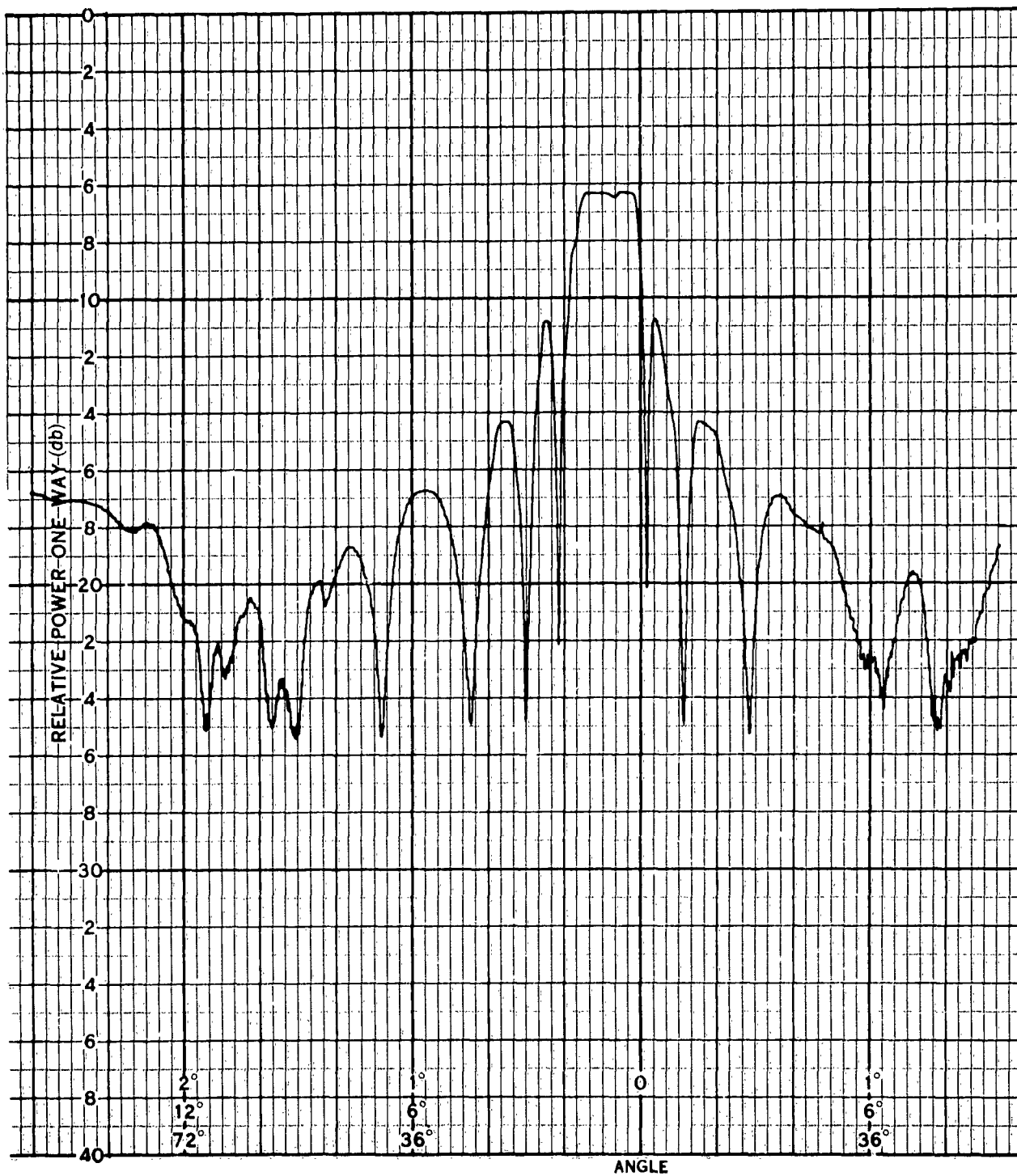


FIGURE 15. RECTIFIED VOLTAGE PATTERN FOR $d = \lambda$
USING THE POLYNOMIAL GENERATOR (LINEAR DETECTION)

side lobe heights and angular spacing agreed well with the theoretical values.

The results of the experiments up to this point showed that the polynomializer did simulate the six-element pattern within a crude degree of accuracy. Thus, the principle was practical, while the refinements required were numerous.

We then proceeded to use portions of our polynomial generator, i. e. the individual power terms to generate patterns such as

$$\left(\frac{\sin x}{x} \right)^3 \quad \text{and} \quad \left(\frac{\sin x}{x} \right)^5, \quad \text{the signal input coming}$$

from our "continuous aperture" slot array. The purpose of this work was to attempt to synthesize more general non-linear operations than were obtained heretofore with the entire polynomializer. We used both linear $\frac{\sin x}{x}$ as well as square law $\frac{\sin^2 x}{x^2}$ detection preceding the cubing and fifth power operations, thus theoretically getting $\frac{\sin x}{x}$, $\frac{\sin^2 x}{x^2}$, $\frac{\sin^3 x}{x^3}$, $\frac{\sin^6 x}{x^6}$, $\frac{\sin^5 x}{x^5}$, and $\frac{\sin^{10} x}{x^{10}}$ patterns. Of these only a few could be realized experimentally in a recognizable fashion. Curve A in Figure 16 shows the pattern of our continuous aperture as detected with a GR 874-VQ Voltmeter Detector, Curve B and C correspond to the voltage of curve A being processed in the cube and fifth power stage of the polynomializer, respectively. We note that the side lobe level of 12 db (13 db theoretically with a square law detector) decreased to 20 db for the cubed pattern, and to about 17-1/2 db for the fifth power pattern. [The (noise) "zero" levels of

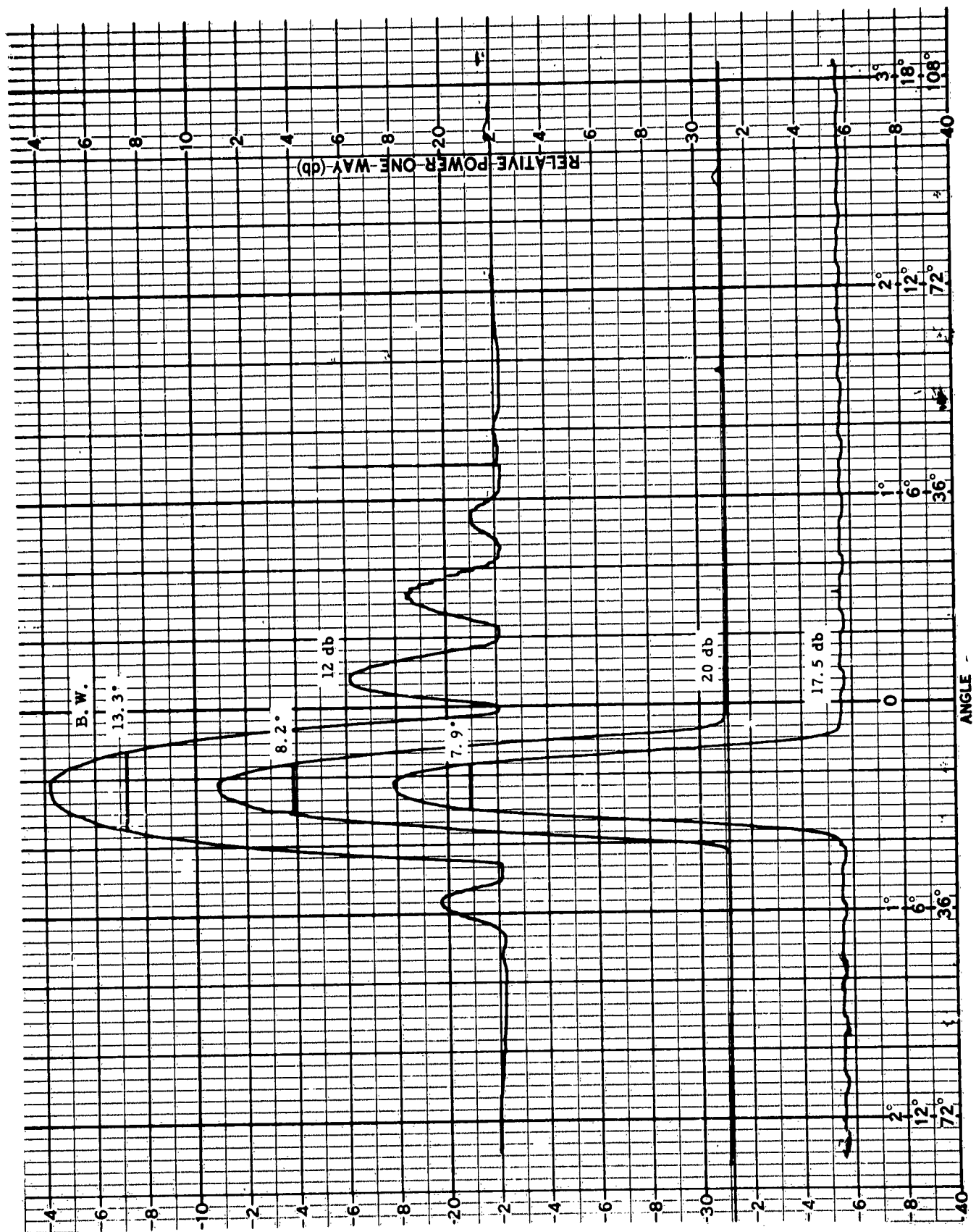


FIGURE 16. CONTINUOUS APERTURE PATTERN SUBJECTED TO POWER RAISING

the patterns were artificially displaced by varying the gain, in order to use a common angular scale for the three patterns.] Theoretically, of course, the side lobe level for the cubed case should have been three times that of the ordinary pattern, i. e., 36 db, while for the fifth power case we should have gotten a 60 db. The considerable falling short of the experimental side lobe level can probably be attributed to the limited dynamic range of the analog power raising devices.

The flat bottoms of the cube and fifth power curves were probably due to the lower threshold of about 1 volt which limited the sensitivity of the analog unit. This, of course, made it impossible to detect a weak signal near the noise level, since the smoothness of the bottoms of the curves indicated that the background noise level was below the analog threshold. A single signal well above the noise would, however, have been easily and non-ambiguously detected with these cube and fifth power patterns. If a device which increases the S/N ratio (e. g. a filter, or an autocorrelator for periodic signals) were to precede the analog power raisers, the apparatus should be able to detect non-ambiguously even signals buried in the noise.

Along a different line of investigation, we built a multiplier to operate at an intermediate frequency of 30 mc. This was done so that the signal amplitude variations due to the frequency varying in and out of the pass-band of the 2 mc second IF could be eliminated. The unit was tested in two set-ups:

The first test was with two horn antennas separated 52 cm on centers. Figures 17 and 18 show the results of these tests. The first figure shows the added pattern at 30 mc. Figure 18 shows the multiplied pattern. The second test was with a continuous aperture and a horn, the horn being located above the last slot in the aperture. The next two figures show the results of this test. Figure 19 is a bolometer pattern of the continuous aperture by itself $\left[\left(\frac{\sin x}{x} \right)^2 \right]$, taken as a reference. Note that the first nulls are approximately 28° apart and the largest side lobe is 13 db down. In the multiplied pattern $\left(\frac{\sin x}{x} \right) \cos x$ in Figure 20 the nulls were 14° apart and the largest side lobe was 6 db down. This side lobe level was reasonable since the first pattern was a power pattern, while the multiplied pattern was really a voltage pattern taken on the same recorder.

It appeared then that the switch from 2 mc to 30 mc multiplication had been worthwhile and that satisfactory independence from oscillator frequency variations had been achieved. Furthermore we had generalized the types of non-linear analog operations available to us, laying the groundwork for future Berman and Clay type experimentation and other operations of interest.

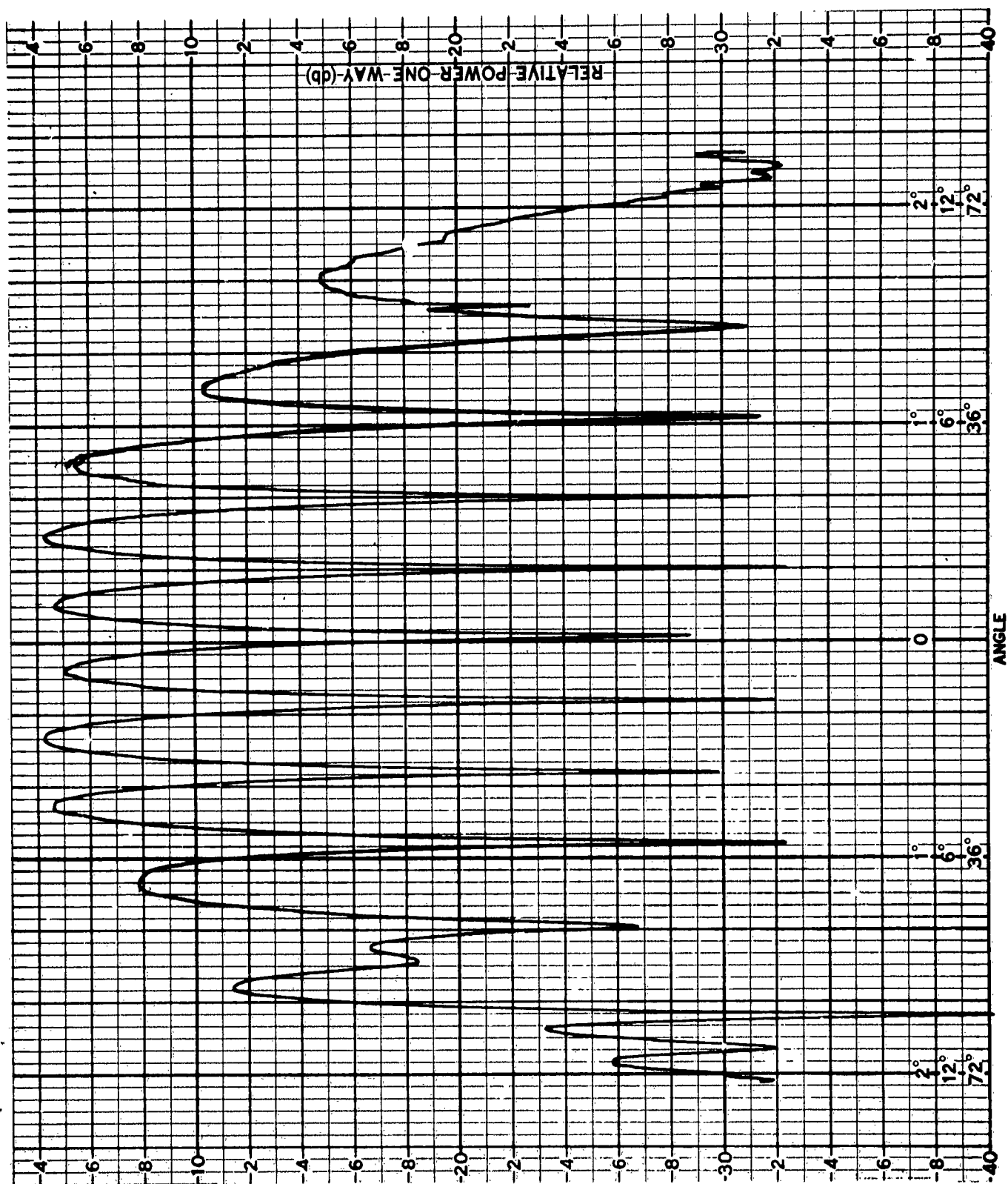


FIGURE 17. TWO HORN ADDED PATTERN

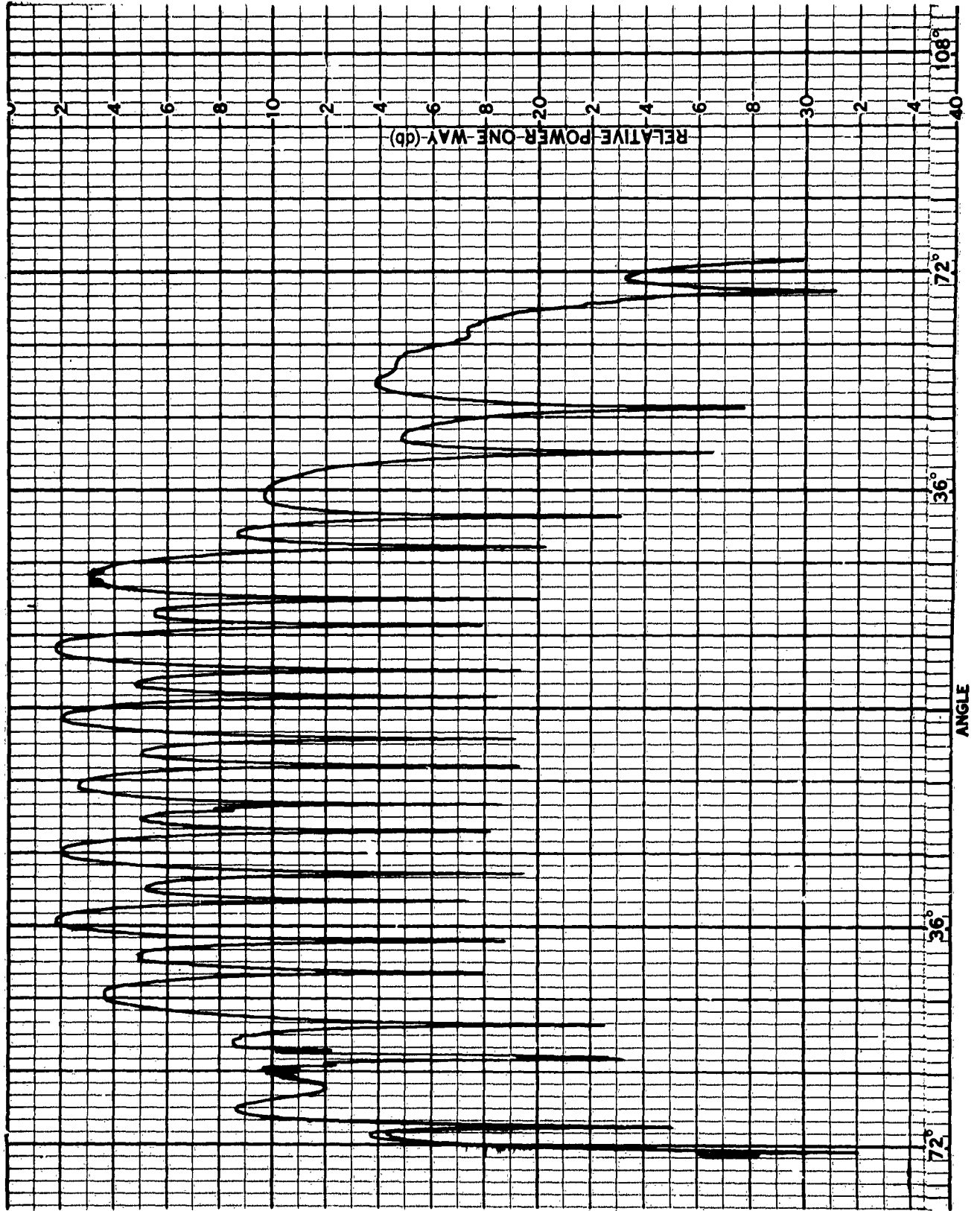


FIGURE 18. TWO HORNS MULTIPLIED AT 30 MC

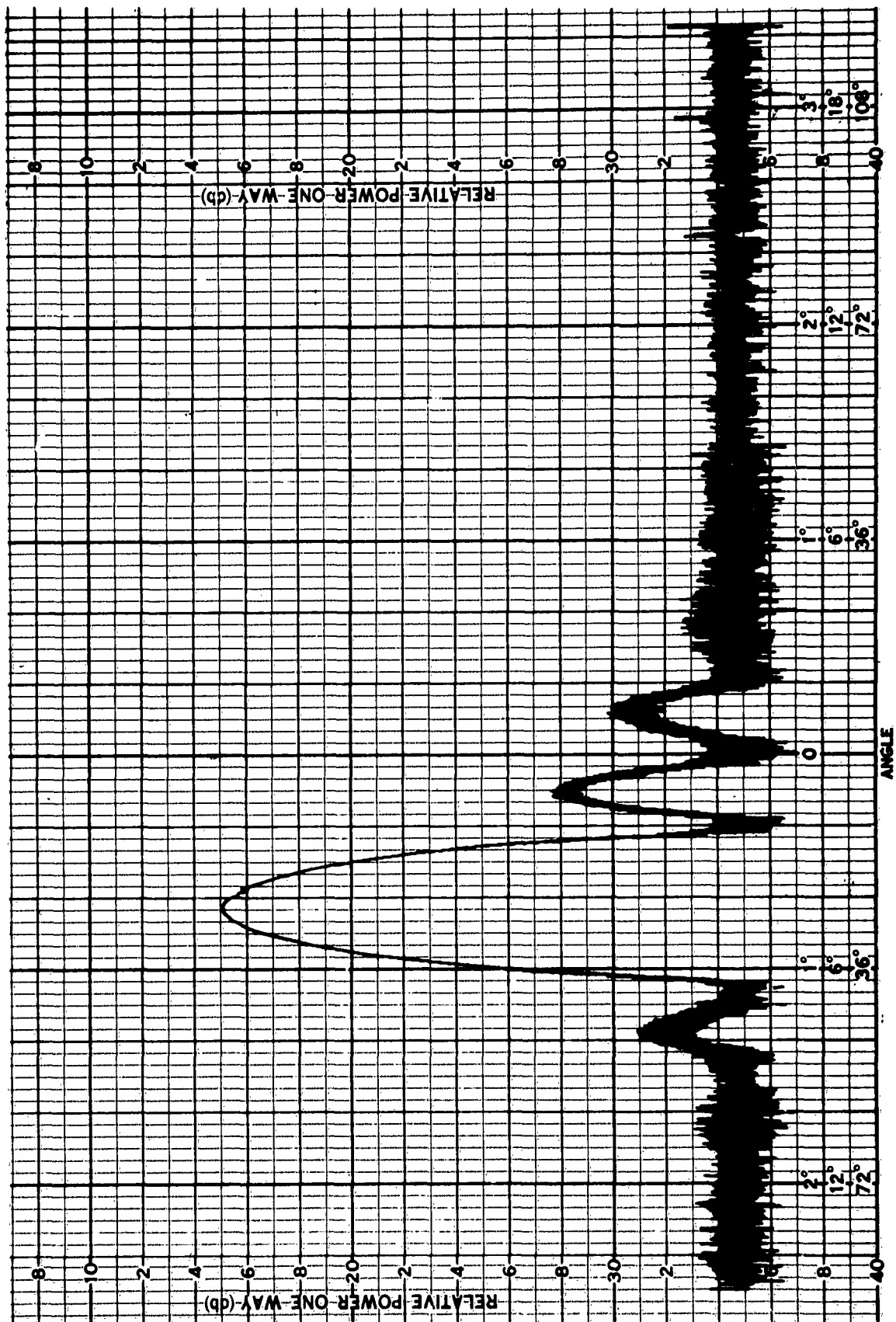


FIGURE 19. BOLOMETER PATTERN OF CONTINUOUS APERTURE $\frac{\sin x}{x}$

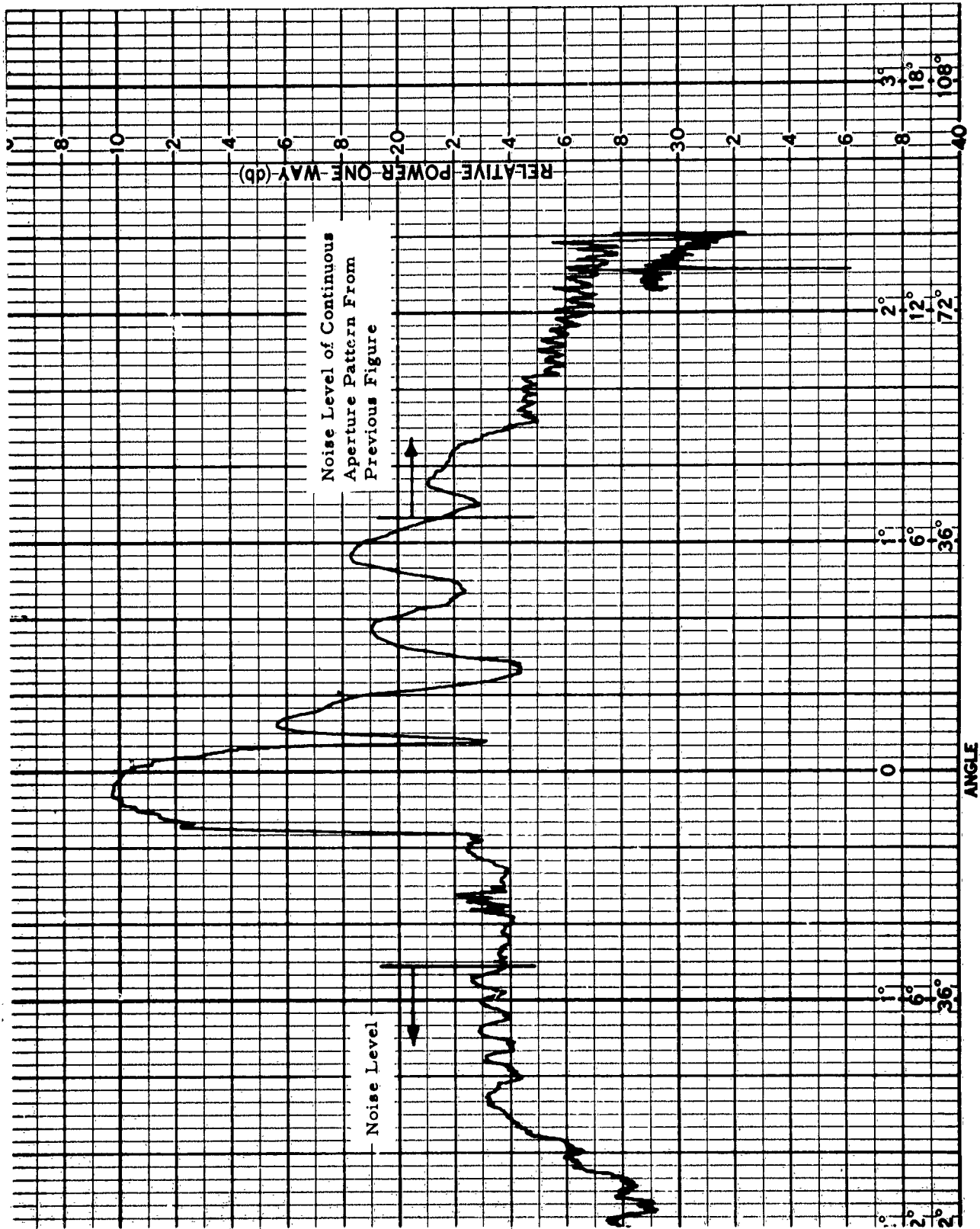


FIGURE 20. PATTERN OF CONTINUOUS APERTURE MULTIPLIED BY SINGLE HORN
 AT ONE END $\frac{\sin X}{X} \cos X = \frac{\sin 2X}{2X}$

Items 2 and 4. Multi-frequency signal correlation and non-uniform aperture illumination

As mentioned previously we found it expedient to combine Work Items 2 and 4 into one effort. The reasons for this are explained in more detail herewith.

We initially designed the following system (see Figure 21):

A slotted waveguide with longitudinal slot spacing $d > \lambda$ was to act as receiving antenna for two signal frequencies, 3060 mc and 3770 mc. Two klystron-fed transmitting horns mounted one above the other on the transmitting tower about 200 feet from the receiving antenna were provided to supply the microwave radiation. Both transmitters were audio-modulated by the same audio oscillator at 1000 cps.

The transverse spacing of the slots of the array was constant, giving an effectively exponential amplitude taper illumination as seen at either output end.

If the angle of a major lobe of such an array is taken as θ , one can derive the following relations between free space wave length λ_{01} , λ_{02} transverse waveguide dimension a , aperture length d , and guide wave lengths λ_1 and λ_2 respectively:

$$\sin \theta_1 = \frac{\lambda_{01}}{\lambda_1} - \frac{\lambda_{01}}{2d} (1-2m) \quad (3)$$

$$\sin \theta_2 = - \left[\frac{\lambda_{02}}{\lambda_2} - \frac{\lambda_{02}}{2d} (1-2n) \right], \text{ where}$$

$$m, n = 0, 1, 2, \dots$$

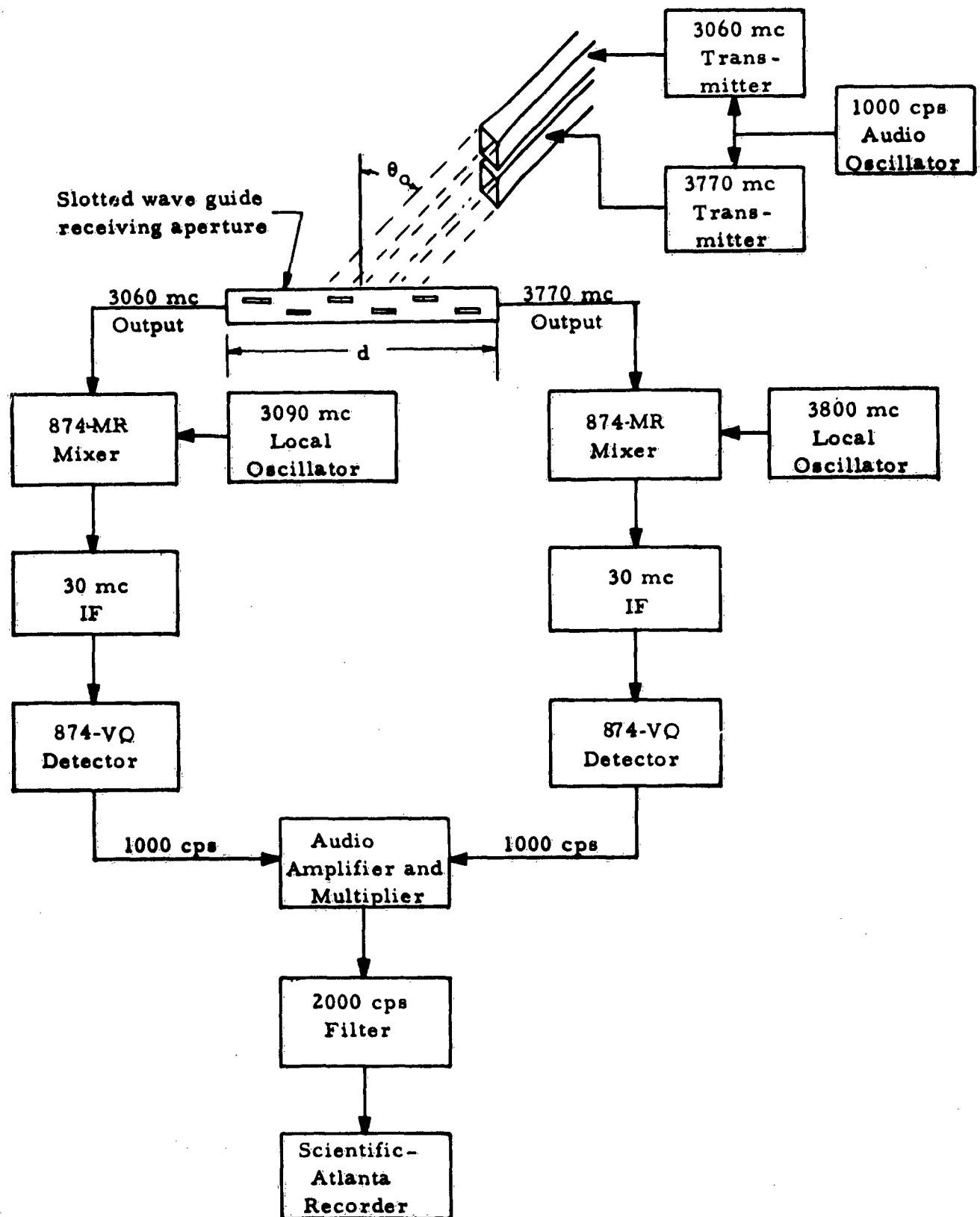


FIGURE 21. TWO-FREQUENCY MULTIPLICATIVE ANTENNA SYSTEM
WITH NON-UNIFORM APERTURE ILLUMINATION

$$\text{and} \quad \lambda_{1,2} = \frac{\lambda_{o1, o2}}{\sqrt{1 - \left(\frac{\lambda_o}{2a}\right)^2}} \quad (4)$$

where the two different frequencies ω_1 and ω_2 will emerge at opposite ends of the waveguide, except for some reflections. In effect the antenna acts as a frequency as well as spatial filter here.

If we now impose the condition $\theta_1 = \theta_2$, i. e., a pair of major lobes from the two patterns of different frequency are made to coincide (at least at their centers) we can now reduce ambiguity if we multiply the patterns. This is due to the non-coincidence of all other pairs of main lobes, and constitutes the frequency domain analog to our former experiments of cross-correlating outputs of two linear arrays of equal frequency but different element spacing.

The coincidence condition $\theta_1 = \theta_2$ can theoretically be achieved by suitable manipulation of the parameters m , n , a and d . We will go into the details of this procedure below when we discuss the system which was actually built and with which we arrived at the multiplicative patterns shown in Figures 29 and 30. (This system differs somewhat from the one outlined above, mainly by the use of two instead of one slotted wave guide).

To continue the overall description of the entire system, the two signals at ω_1 and ω_2 are both heterodyned to 30 mc, detected and then multiplied in a specially designed AF multiplier whose schematic appears in Figure 22.

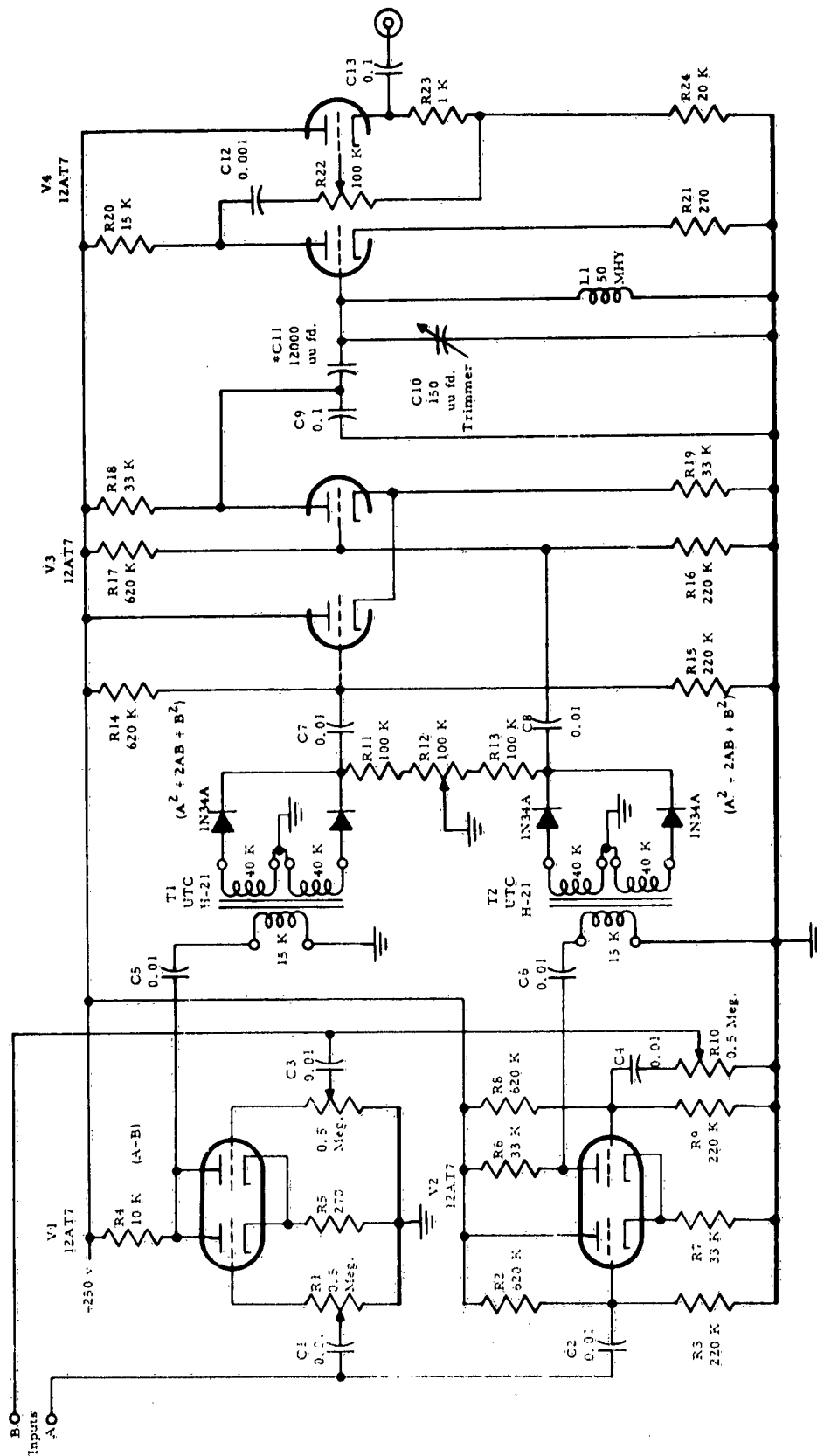


FIGURE 22. AUDIO FREQUENCY MULTIPLIER INCORPORATING AMPLIFIER AND 2000 CPS FILTER

The design differs from that previously used by us to correlate 2 mc and 30 mc signals, with the substitution of audio-frequency components wherever needed, such as the three transformers, but more importantly, by using the "Quarter-Square-Difference" multiplication principle, as was done in work performed on the preceding contract.

The 2000.cps output whose amplitude is proportional to the amplitude product of the two 1000 cps inputs is filtered and fed directly to the per amplifier stage of a Scientific-Atlanta pattern recorder by passing its internal filter. Note that a stage of audio amplification at each input forms part of the multiplier design.

Note that the present signal-processing scheme differs significantly from those previously promulgated under this and the preceding contract. We are now cross-correlating (with time delay always zero) two coherent audio frequency signals whose amplitudes are proportional to the pattern functions of their respective carriers. They are equal in frequency and phase since they are derived from a common audio modulator at the transmitters, but differ in general in amplitude. The incoherence of the two carriers and local oscillators has no effect here.

Previously we correlated two modulated RF voltages which were coherent for any given direction of the incident plane wave. To maintain this coherence we were forced to use a common local oscillator.

Going back to the design of the present experiment, we required for overlapping of at least one pair of main lobes that the guide wave lengths λ_1 , λ_2 and the free space wave lengths λ_{01} , λ_{02} of the two

signals satisfy the equation

$$\frac{\lambda_{01}}{\lambda_1} - \frac{\lambda_{01}}{2d} (1 + 2m) = - \frac{\lambda_{02}}{\lambda_2} + \frac{\lambda_{02}}{2d} (1 + 2n) \quad (5)$$

$m, n = \text{positive integers}$

for a slotted waveguide array of longitudinal slot spacing $d > \lambda_1, \lambda_2$.

This equation follows immediately from equation (3) given previously.

For a single slotted waveguide d has only one value, and this is given by

$$d = \frac{\lambda_{02} (1 + 2n) + \lambda_{01} (1 + 2m)}{2 \left(\frac{\lambda_{01}}{\lambda_1} + \frac{\lambda_{02}}{\lambda_2} \right)}, \text{ solving (5) for } d. \quad (6)$$

We tried several combinations of values for the ratios $\frac{\lambda_o}{\lambda}$

and the integers m and n , in the general frequency region of about 2800 -

4200 mc, but no accurate overlapping of main lobes could be achieved

except for the frequencies 3075 mc and 3925 mc which gave an overlap

match of $20^\circ .08$ ($n = 2$) versus $19^\circ .94$ ($m = 1$), i. e. within about $8-1/2^\circ$.

These two frequencies, however, are sufficiently far apart that we could

not receive both of them with the same array, as its bandwidth was about

2%. We therefore resorted to building two slotted waveguide arrays,

each tuned to its respective frequency, sharing all dimensions except the

slot length. The shop drawings of the two waveguides are given in

Figures 23 and 24. The general arrangement of the resulting system is

depicted in the block diagrams of Figures 25 and 26. Figure 25 is self-

explanatory. Figure 26 shows the matched loads attached to the two

arrays fed at oppositely* directed ends. Even though the relatively

The physical reversal of the waveguide ends was also necessitated by the overlap condition (2).

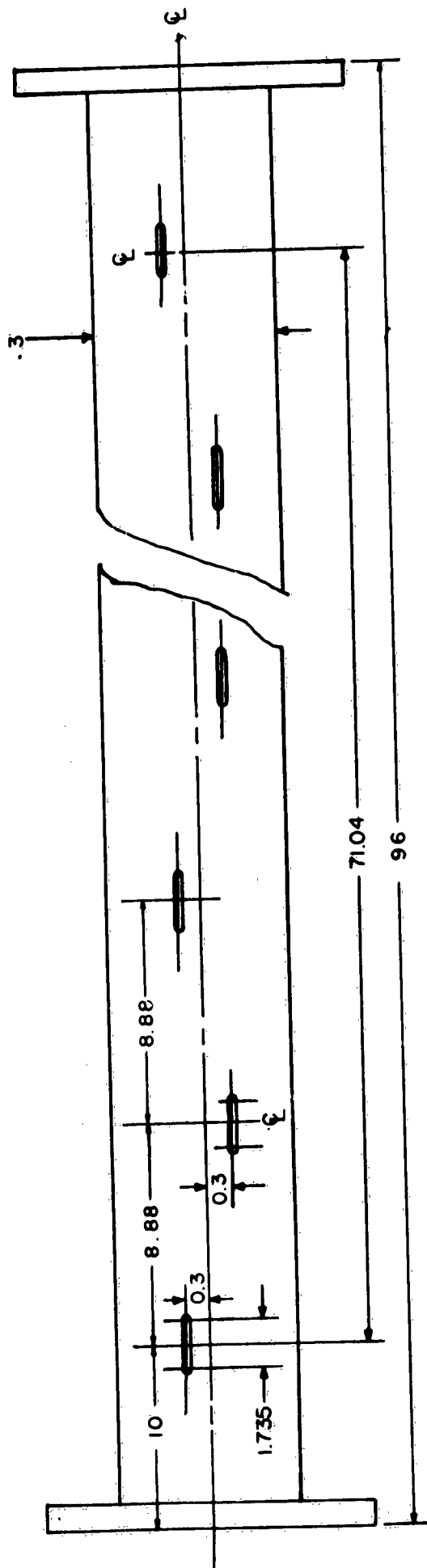
Z

f=3075 MC

UG-53/U

S-BAND GUIDE RG-48/U 1- x 3 O.D.

COVER FLANGE UG-53/U



NOTES:

1. NINE SLOTS EQUALLY SPACED WITH RESPECT TO EACH OTHER AND THE CENTER LINE OF WAVEGUIDE

2. FACE OFF COVER FLANGE AND WAVEGUIDE

3. SLOT LENGTH 1.860 IN.

4. SLOT WIDTH 0.125 IN.

(All Dimensions in Inches)

FIGURE 23. DIMENSIONS OF 3075 MC SLOTTED WAVEGUIDE

COVER FLANGE UG-53/U

S-BAND GUIDE RG-48/U $1\frac{1}{2} \times 3$ O.D.

UG-53/U

3

10

9.335

1.330

0.3

0.3

8.88

8.88

71.04

96

96

1. NINE SLOTS EQUALLY SPACED WITH RESPECT TO EACH OTHER AND CENTER LINE OF WAVEGUIDE

(All Dimensions in Inches)

3. SLOT WIDTH 0.125"

FIGURE 24. DIMENSIONS OF 3925 MC SLOTTED WAVEGUIDE

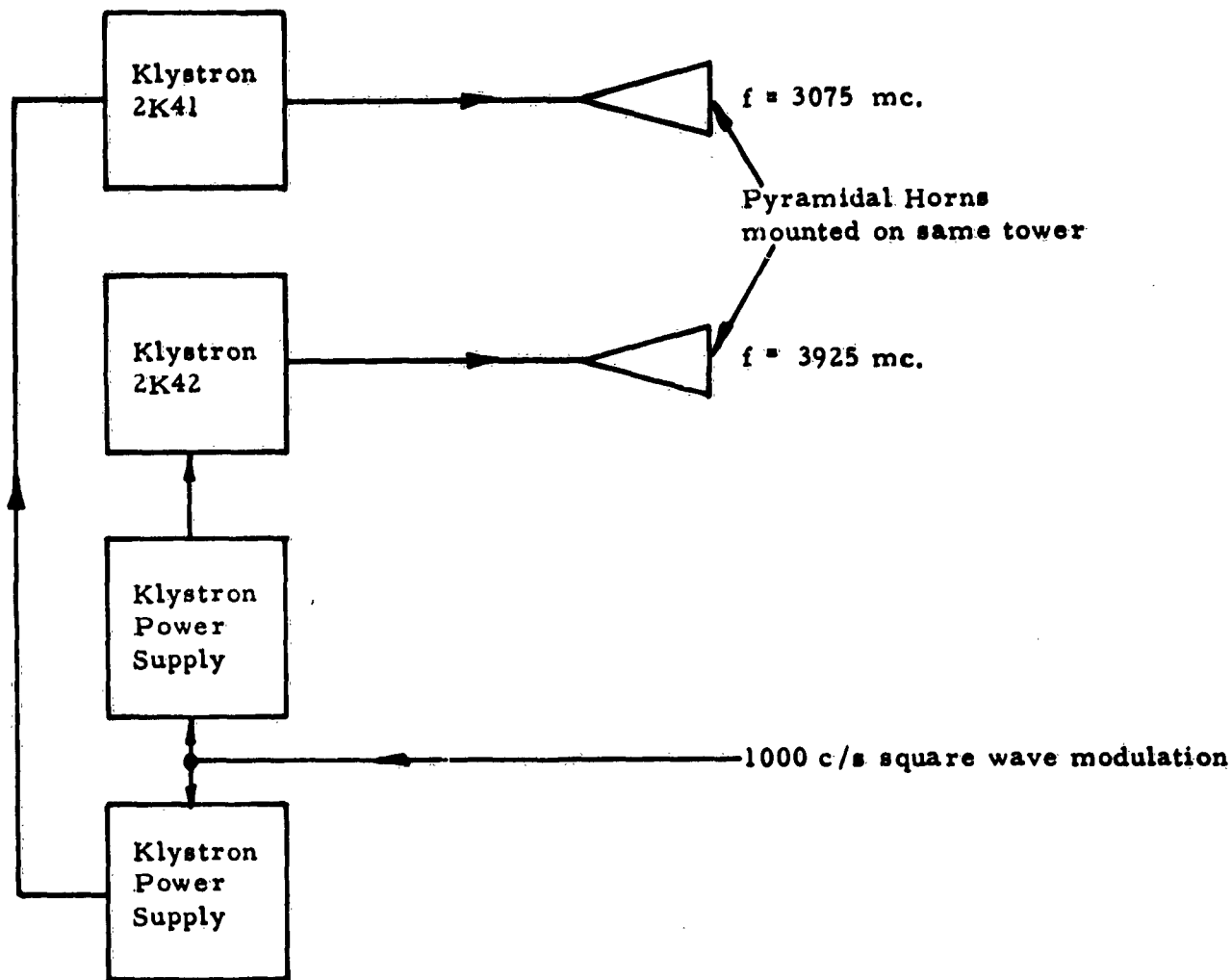


FIGURE 25. TRANSMITTER SETUP

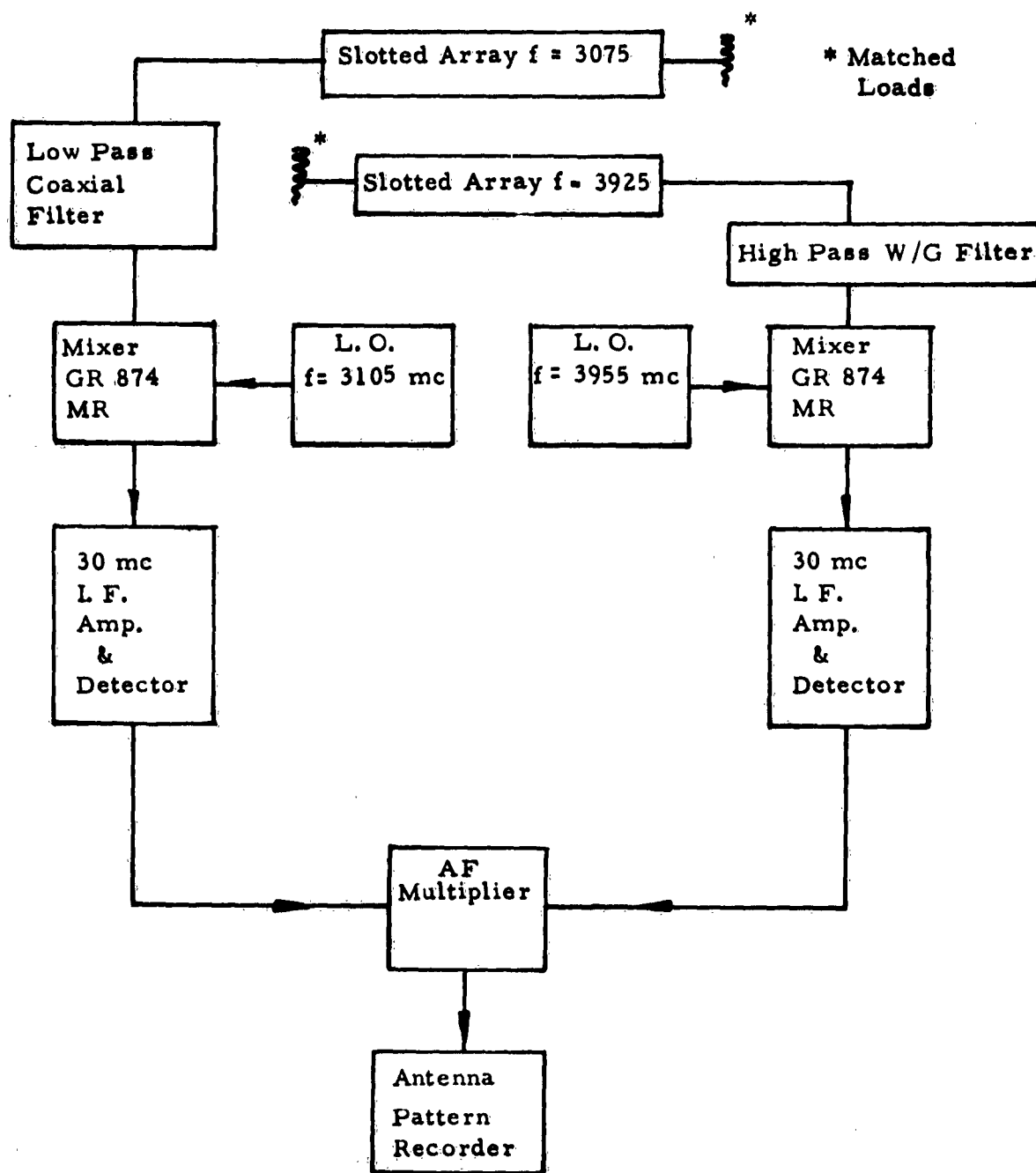


FIGURE 26. RECEIVER SETUP

limited bandwidth of the waveguides causes them to act as natural band pass filters, we decided to insert a high-pass waveguide filter and a low-pass coaxial filter, ** respectively, in the output lines to improve signal separation still further. The multiplication at the end is carried on in the specially designed AF multiplier, previously discussed (Figure 22) which incorporates a 2000 cps filter. The Scientific-Atlanta Recorder is fed at the Pen Amplifier input with the resultant 2000 cps signal.

The results of the double-frequency experiment are shown in the patterns of Figures 27, 28, 29, 30 and 31. Figures 27 and 28 show the pattern of the individual slot arrays, where crystal detection takes place at RF, eliminating the IF stages. Note that no bolometer was used here, but the signal was still fed into the recorder preamplifier. The figures exhibit the usual ambiguous $\frac{\sin N x}{N \sin x}$ type patterns, with a side lobe level of about 14 db at 3075 mc and about 11-1/2 db at 3925 mc. These values are thus reasonably close to the theoretical value of 13 db. (Note that the usual "square-law" detection was employed).

In Figures 29 and 30 we again have the individual array patterns, but this time the signal passes through the IF stages and video detectors and is then fed directly to the pen amplifier of the pattern recorder. The side lobe levels here are about 7 db at 3075 mc and close to 8 db at 3925 mc which would indicate that the detectors are here operating pretty much in their linear response region (they have sufficient drive signal due to the high IF gain), whereas in the patterns of Figures 27 and 28 the detectors

** These were the only two workable filters available at the time.

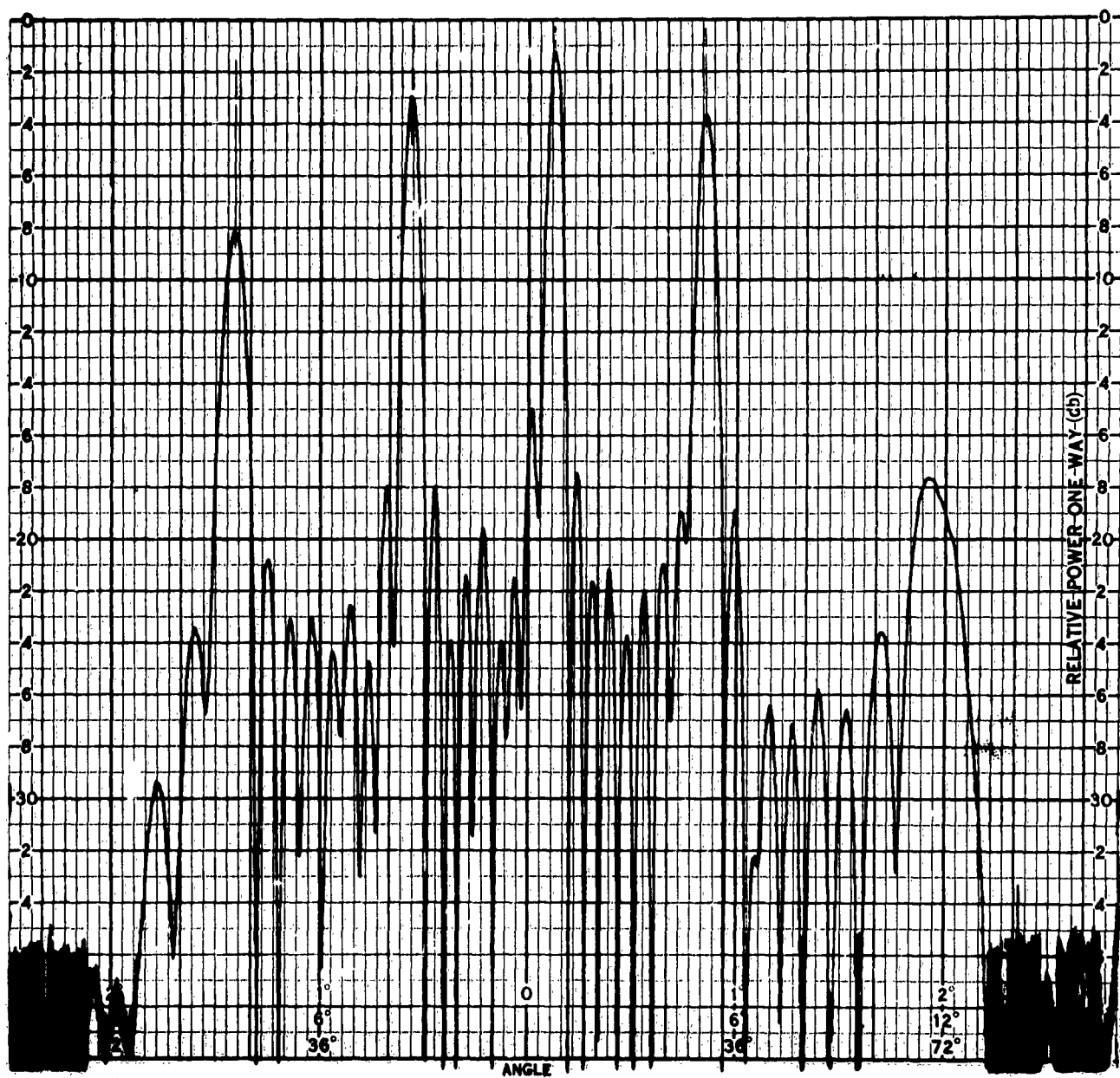


FIGURE 27. PATTERN OF 3075 MC SLOTTED WAVEGUIDE ARRAY,
CONVENTIONAL CRYSTAL DETECTION, NO IF

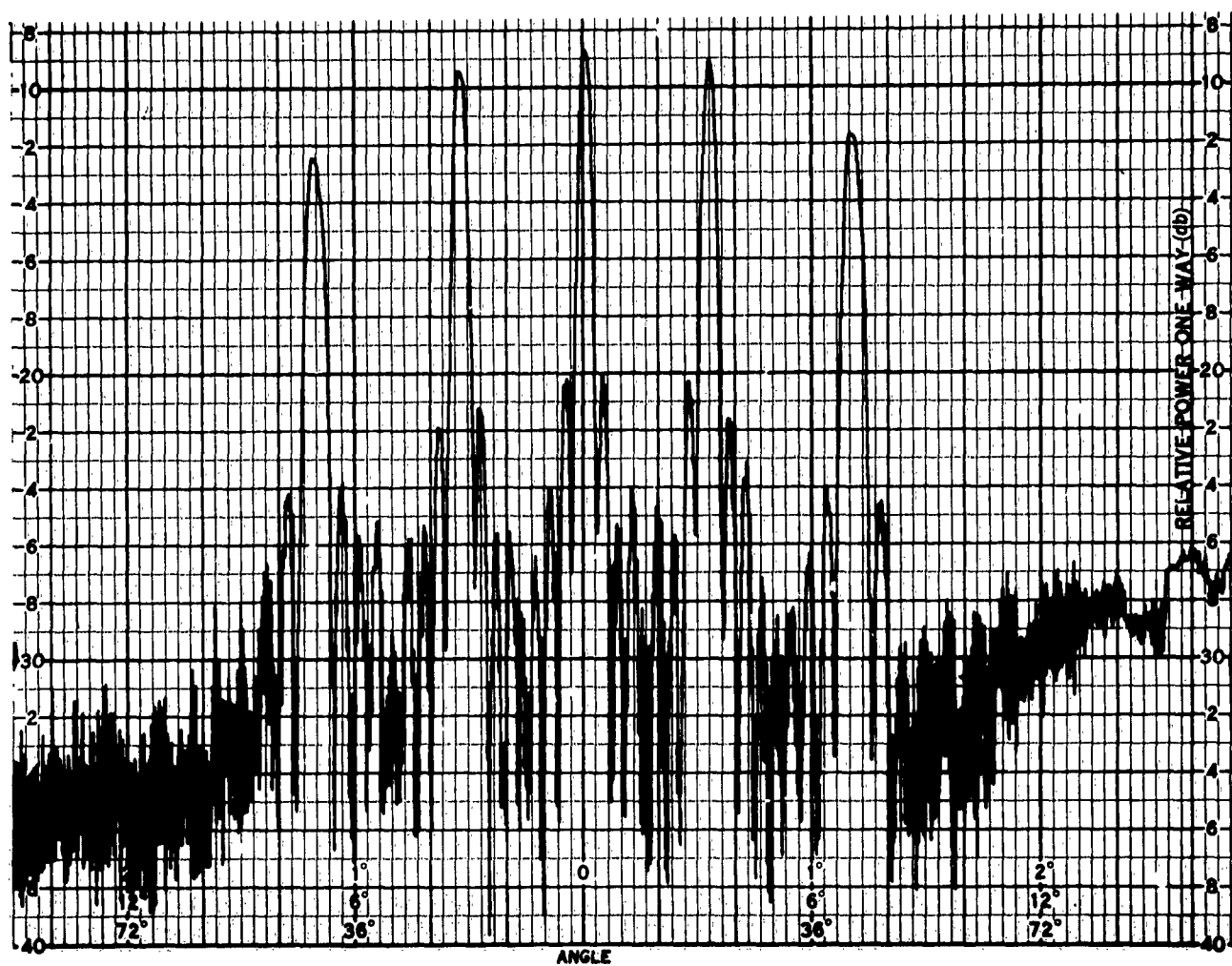


FIGURE 28. PATTERN OF 3925 MC SLOTTED WAVEGUIDE ARRAY,
CONVENTIONAL CRYSTAL DETECTION, NO IF

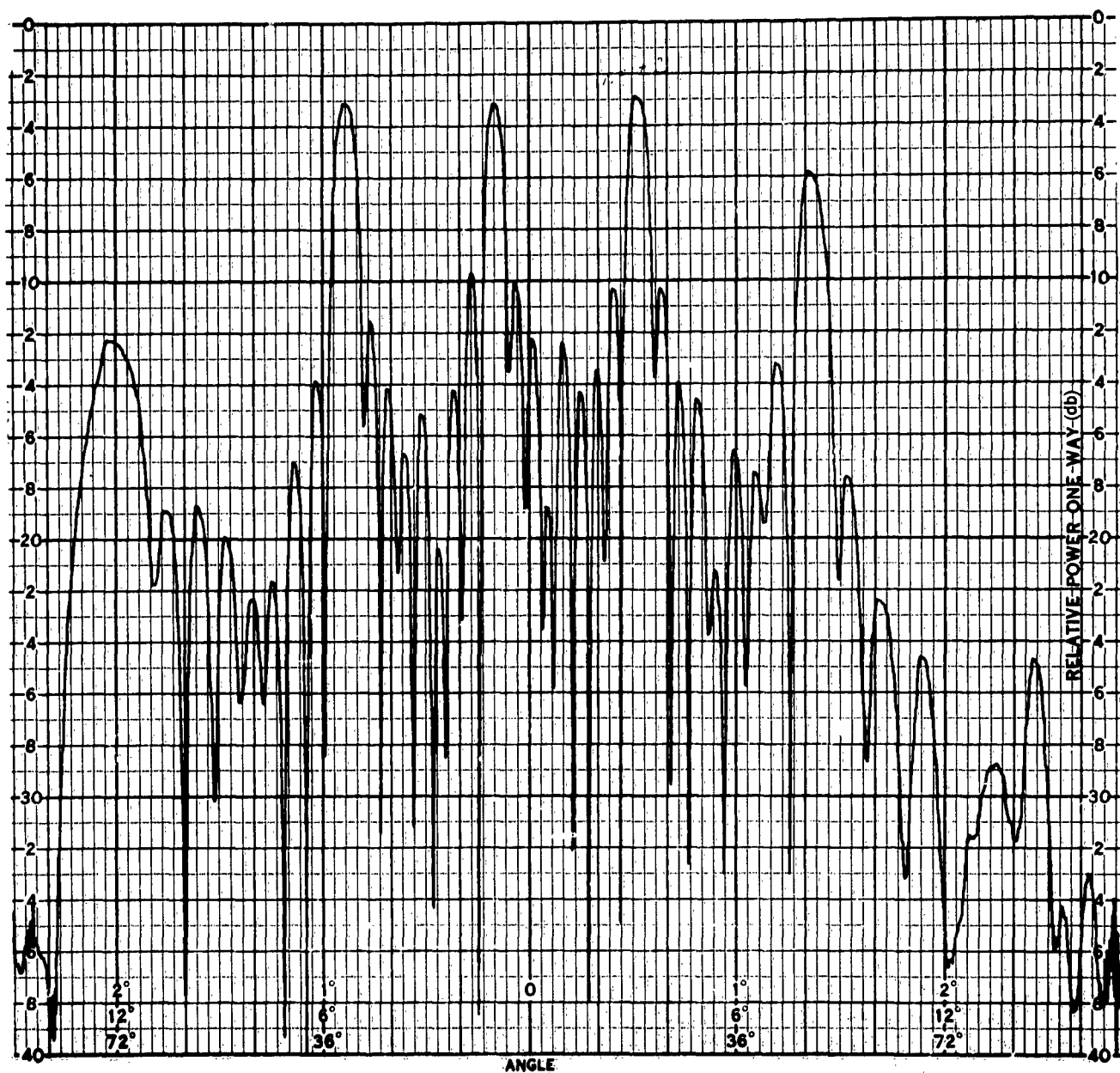


FIGURE 29. PATTERN OF 3075 MC SLOTTED WAVEGUIDE, 1000 CPS MODULATION. VIDEO OUTPUT OF IF AMPLIFIER FED DIRECTLY TO RECORDER PEN AMPLIFIER

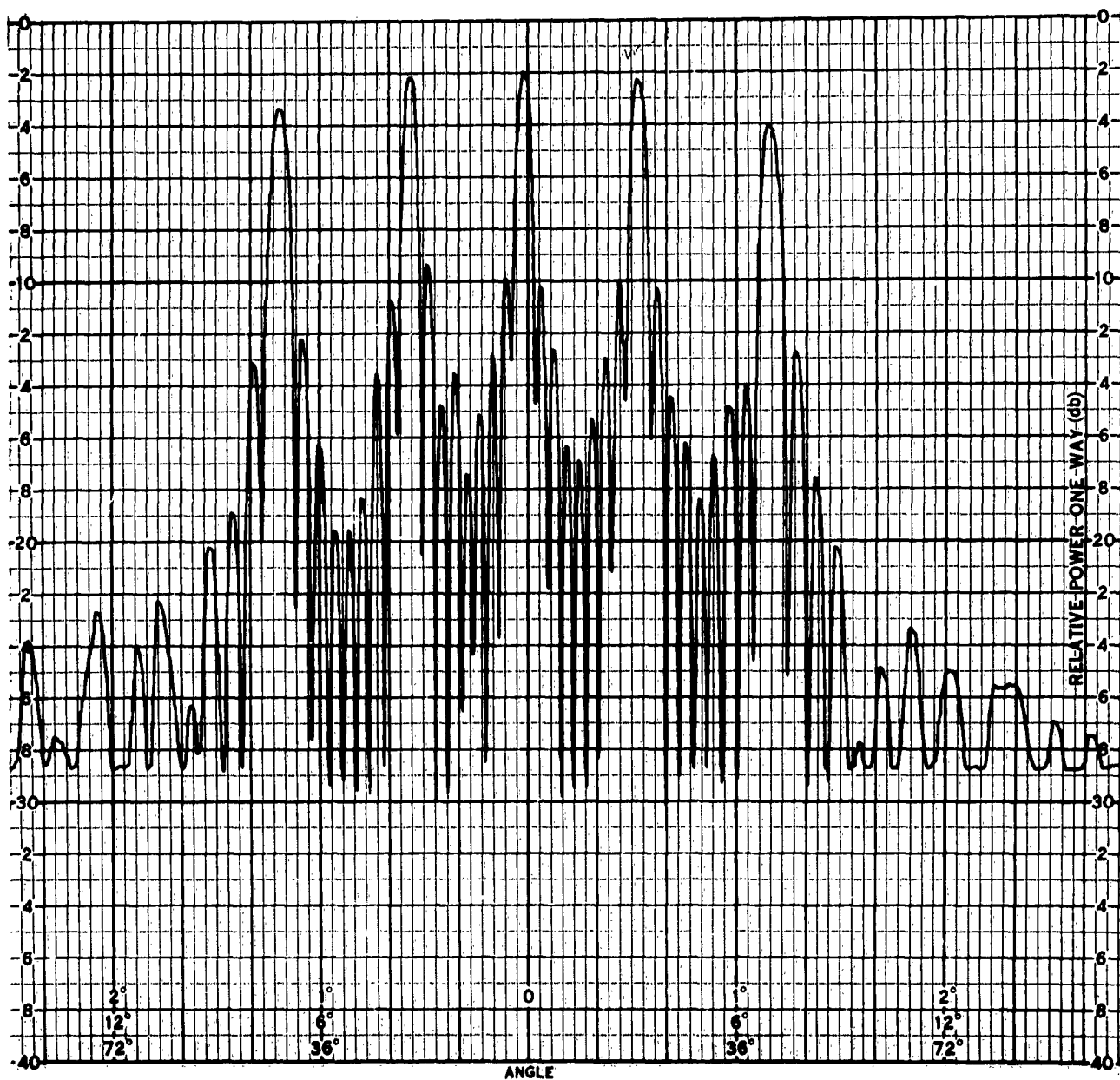


FIGURE 30. PATTERN OF 3925 MC SLOTTED WAVEGUIDE, 1000 CPS MODULATION. VIDEO OUTPUT OF IF AMPLIFIER FED DIRECTLY TO RECORDER PEN AMPLIFIER

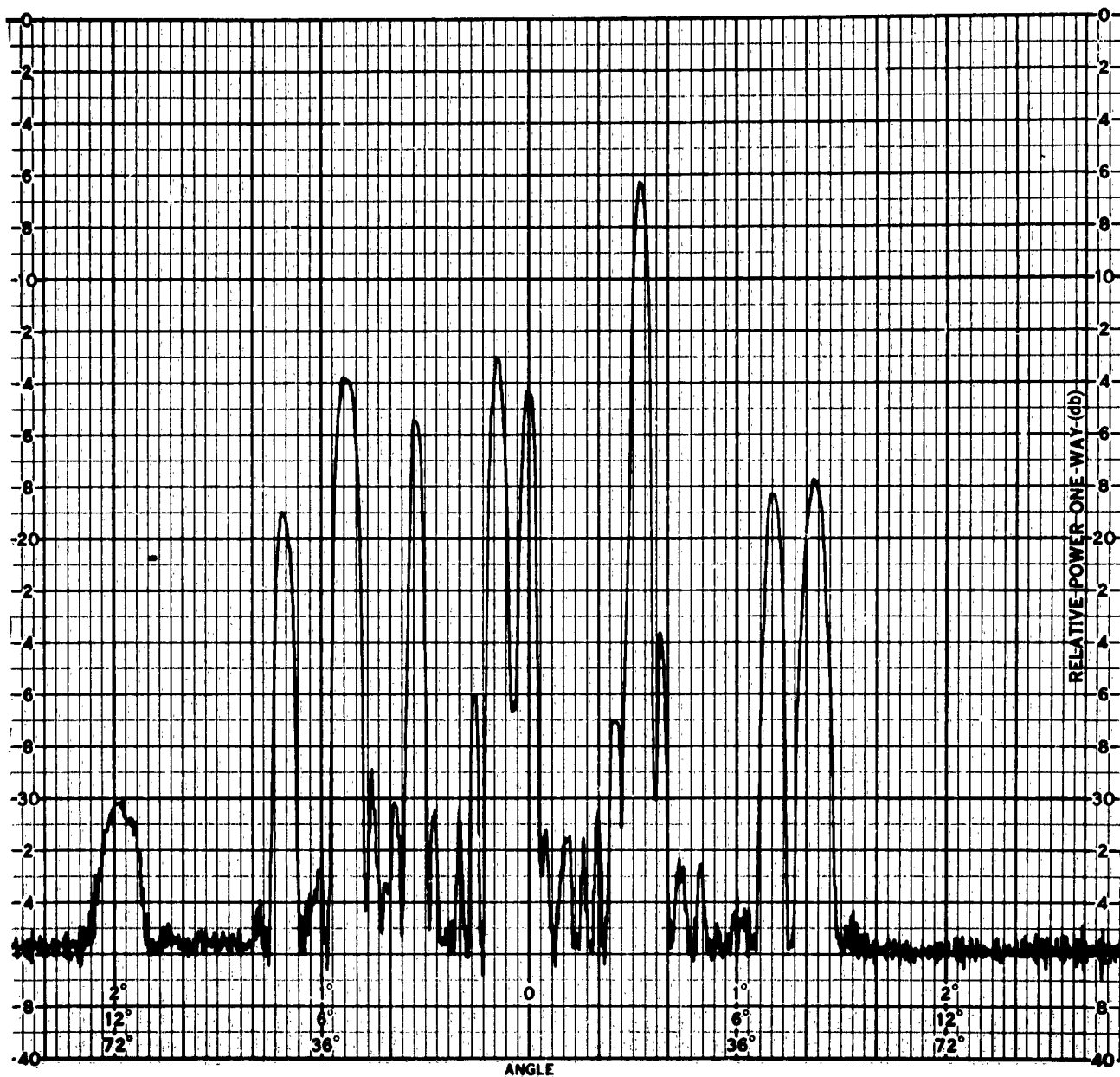


FIGURE 31. MULTIPLIED PATTERN

probably operated in their square-law regions due to the low drive voltage of our S-band local oscillators.

After we multiplied the detected 1000 cps voltages and filtered for 2000 cps we obtained the pattern shown in Figure 31. One can clearly see the enhancement of the overlapping multiplied main lobes at approximately 19° azimuth. (All pattern charts are to be read with the bottom one of the three angular scales). The removal of ambiguity is clearly shown by the $6\text{-}3/4$ level difference between the enhanced main lobe and the highest non-enhanced one.

The results thus show the feasibility of producing correlation-type patterns at multiple frequencies, using essentially one array. Even though we were forced to use two (very similar) arrays in this experiment single array with more broadband elements (say, a set of wave guide horns connected into a corporate structure) can readily be designed to enable one to carry out the experiment as initially planned.

Finally we wish to discuss the question of non-uniform aperture illumination, as specified by Item 2 of the Work Statement. As stated previously, we had hoped to design a single slotted waveguide array which would operate at two distinct frequencies and which would act as an intrinsic filter, providing one signal frequency at one end and the other frequency at the opposite end. Because of the "double-ended" feed method, it was necessary to provide an aperture illumination which was symmetrical about the array bisector, in order not to discriminate spatially between the two signal frequencies. The simplest such

illumination to design is the exponential one, i. e. constant transverse spacing of the slots. This was therefore used, resulting in oppositely directed exponential aperture illuminations for the two frequencies. When it was decided that two arrays instead of one were to be built, we used the array at hand for 3075 mc and built another similar one resonant at 3925 mc. Thus it was that the optimized illumination required under Item 2 came to be an exponential one for this case, even though we were unable to vary the illumination function.

We define an "optimum" illumination here as one obtainable by a simple mechanical design, together with acceptable sidelobe level and beam width of the resultant pattern.

Item 3. Theoretical concepts of gain, directivity and system efficiency for non-linear processing

Along theoretical lines, we gave considerable thought to the problem of antenna gain, directivity, and antenna efficiency as applied to a non-linear system. Here the antenna proper could not be considered apart from the associated non-linear circuitry and it was necessary that commonly-used and time-tried antenna parameters, such as gain, directivity, etc. be re-examined and perhaps redefined from a basic point of view, since they had been defined in terms of electrically linear antenna systems heretofore. Another principle, that of reciprocity between transmitting and receiving antennas, cannot hold for non-linear systems and had to be abandoned.

Our attention was initially concentrated on the interrelated parameters antenna gain, directivity, resolution, and radiating efficiency. In ordinary linear systems three of these parameters are related by the simple equation

$$G = kD \quad (7)$$

where:

G = antenna gain

k = radiation efficiency

D = antenna directivity

and the antenna is assumed to be perfectly matched. In non-linear

systems, however, resolution* can obviously be enhanced by certain non-linear operations, without necessarily increasing the directivity and/or gain. A simple example may serve as an illustration. If we raise the $\frac{\sin x}{x}$ voltage pattern from a continuous aperture to some power n, we obviously get a pattern of the form $\frac{\sin^n x}{x^n}$ whose main lobe is much narrower than that of the original $\frac{\sin x}{x}$ pattern**. Therefore, the resolution, if defined in terms of half-power point beam width has increased.

On the other hand, no additional physical power-gathering capacity can be imparted to the antenna by the non-linear signal processing circuitry and hence, the gain need not have increased. This means that we must assume that either the radiating efficiency k is now a function of the non-linear circuitry operations if relation (7) is to continue to hold, or that directivity along with gain now depend on resolution in a different way than for linear antenna systems. It is probable that both suppositions are true. Note also that, if resolution is defined a la Rayleigh, i. e. in terms of location of pattern nulls and maxima at the main beam, then even the resolution has not changed in our simple example of raising the $\frac{\sin x}{x}$ pattern to some power n. It became pretty obvious that our first task in the further theoretical treatment of non-linear antenna systems was to construct a set of carefully defined generalized antenna system parameters.

* as given by half-power beam width

** even though its nulls and the maximum height are the same as for $\frac{\sin x}{x}$.

Reviewing our task we see that the questions to be answered were:

a) Can we formulate consistent expressions for directivity, gain, and efficiency of such non-linear systems, analogous to these properties as used with linear systems?

b) What effective "antenna pattern" is produced at the output terminals of the system?

c) Is there an equivalent electrically linear antenna system which would produce the same pattern?

Even though only problem a) appeared formally in the Work Statement, we realized that problems b) and c) needed to be studied concurrently.

In order to be completely general, let us assume that the system consists of a (linear) antenna of pattern $f(x)$, a "black box" containing the signal processing circuitry, and a recorder which responds to the db output of the black box. (See Figure 32)

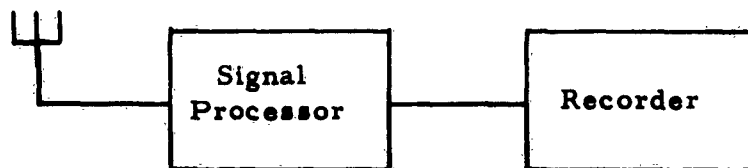


FIGURE 32. BASIC NON-LINEAR RECEIVING SYSTEM

We will formulate our ideas in such a way that the performance of the ordinary square-law detector type receiving system (using diodes, bolometers, etc.) forms a special case whose behavior is known from experience and can be used to test the validity of the proposed formulation.

1. Directivity, Gain, and Efficiency

These parameters are simply related for electrically linear antenna systems. We have in that case: $G = \alpha D$ (8)

G = Gain

D = Directivity

α = Effectiveness ratio

The factor α involves both radiating efficiency and mismatch effects. Since we will assume perfect match for the purpose of this discussion, it becomes equal to the efficiency ($0 \leq \alpha \leq 1$).

The gain has here been referred to a lossless isotropic antenna (See Antennas, J. D. Kraus, McGraw-Hill, 1950).

$$\text{One also has } D = \frac{4\pi f(\theta, \phi)_{\max}}{\iint f(\theta, \phi) d\Omega} \quad (9)$$

where the radiation intensity (power density) as a function of polar angles

θ and ϕ is given by $f(\theta, \phi)$, and the double integration is over the entire solid angle 4π .

We note that for conventional (electrically) linear antenna systems, all of the power is accounted for in some way, as useful power, reflected power, losses, etc. In a generalized system using non-linear processing

of the antenna signals, there are "outside" contributions to the signal power from, say, amplification, as well as losses. It seemed clear that a gain or efficiency definition should not follow the method of the power engineer who has to account for all of his power, but should conform to the needs of the communications engineer who is more interested in signal-to-noise ratio, distortion, intelligibility, angular discrimination, and other "relative" quantities. We therefore propose that directivity, gain, and efficiency simply be defined by the properties of the signal at the output terminals of the non-linear (and still undescribed) circuitry (this may be lumped into a "black box") connected to the antenna, independent of the operations performed on the signal in the "black box." As a reference system we may then use an isotropic antenna with a black box having unit gain and zero phase shift (unity transfer function).

We also had to recognize that antenna parameters are conventionally defined with the implicit assumption of a power (square-law) detector system being present. To make our new definitions further agree with the old ones we propose to continue to use the "power" $f(\theta, \phi)$ (i. e. output voltage squared) at the output terminals of the system as the independent variable in the angular power pattern (directivity). This "power" is then directly proportional to that gathered by the equivalent linear antenna system (see c) above).

It is clear that the "effectiveness ratio" now becomes a function of the internal efficiency and external mismatch of the black box (among

other factors), in addition to the radiating efficiency and mismatch of the antenna proper. We therefore conclude that directivity is the most significant quantity describing such non-linear systems, since it remains independent of the rather meaningless efficiency and mismatch properties of the "pattern-shaping" black box.

We also had to account for the effect of changes in signal-to-noise ratio and distortion introduced by the black box. These processes can influence the shape of the directivity pattern and hence should also appear as independent variables governing the effectiveness ratio

Summing this up we may say that it appears that we can retain the elementary relation $G = \alpha D$, provided the variables are generalized as stated above, particularly the efficiency which can be made a function of several appropriate system parameters. This does not of course imply an intrinsic necessity for retaining the simple relation between G , and D .

If we do assume that (8) holds, the effectiveness ratio α can be analyzed quantitatively as to its probable functional dependence on, say, the transfer function of the "black boxes", their noise figure, external mismatch and gain. Signal-to-noise ratio and distortion will then be automatically accounted for.

For ordinary linear systems* we have for the "effectiveness ratio" α

$$\alpha = \frac{A_e}{A_{em}} \quad (1)$$

* See J. D. Kraus, "Antennas," McGraw-Hill, 1950.

where

$$A_e = \text{effective aperture} = \frac{w(\text{power in terminating impedance})}{p(\text{power density of incident wave})}$$

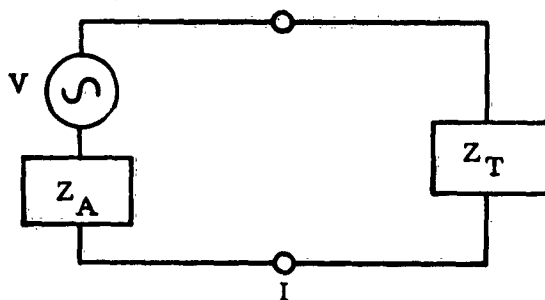
$$A_{em} = \text{maximum } A_e = \frac{V^2}{4PR_r} \quad (2)$$

where R_r = radiation resistance

V = equivalent (Thevenin) voltage of antenna.

If the equivalent circuit of the antenna is taken as in Figure 33

below,



$$Z_A = R_A + jX_A = \text{Antenna Impedance}$$

$$Z_T = R_T + jX_T = \text{Terminating Impedance}$$

FIGURE 33. EQUIVALENT ANTENNA CIRCUIT

we may write from (1) and (2)

$$a = \frac{W}{P} = \frac{4PR_r}{V^2} = 4R_r \frac{W}{V^2}$$

$$\text{and since } W = I^2 R_T, \quad I = \frac{V}{|Z_T + Z_A|}$$

$$W = \frac{V^2 R_T}{|Z_T + Z_A|^2} \quad \text{and} \quad (3)$$

$$= \frac{4R_r R_T}{|Z_T + Z_A|^2}$$

The antenna resistance R_A can be split into radiation and loss resistance

$$R_A = R_r + R_L, \text{ giving us}$$

$$a = \frac{4R_r R_T}{(R_T + R_r + R_L)^2 + (X_T + X_A)^2}, \quad (4)$$

In the event of perfect match,

$$R_T = R_r, \quad X_T = -X_A \text{ and}$$

$$\begin{aligned} a &= \frac{4R_r^2}{(2R_r + R_L)^2} = \left(\frac{2R_r}{2R_r + R_L} \right)^2 = \\ &= \left(\frac{1}{1 + \frac{R_L}{2R_r}} \right)^2 = k \text{ (radiating efficiency)}. \quad (5) \end{aligned}$$

Returning to the general expression (4), we see that in a non-linearly processed antenna system, one generalization of the expression for amounts to considering the non-linear processors together with the antenna proper as an equivalent voltage source V in series with an equivalent "antenna impedance" Z_A , terminated by a detecting device of impedance Z_T . Of course this involves writing an equivalent circuit for the entire antenna - non-linear processor system and evaluating its Z_A , but this is a routine circuit theory procedure. Amplifier gains, noise figures, etc. can be included in the equivalent circuit in the form of "transformers," active noise voltage sources, etc. respectively. By making some of the lumped constants of the equivalent circuit non-linear, we can

account for any non-linear behaviour of a signal processor. For example, a cubing device of gain G could be represented as an ideal transformer of turns ratio $G^{1/3}$, whose secondary is shunted by a constant resistor R in series with a non-linear one of characteristic $R_{NL} = R_o I^{-2/3}$, I being the current in the secondary. If $R \ll R_{NL}$, the output voltage across R , $e_o = IR$, will be

$$I = \frac{e_i G^{1/3}}{R + R_{NL}} \approx \frac{e_i G^{1/3}}{R_{NL}} = \frac{e_i G^{1/3}}{R_o} I^{2/3} \quad (6)$$

$$\frac{e_i G^{1/3}}{R_o} = I^{1/3} \quad \text{for input voltage } e_i.$$

Since $I = \frac{e_o}{R}$,

$$\frac{e_i G^{1/3}}{R_o} = \left(\frac{e_o}{R} \right)^{1/3} \quad (7)$$

$$e_o = \frac{RG}{R_o^3} e_i^3 \quad \text{which has the desired transfer function.}$$

For an equivalent multiplier circuit, one could start from the "quarter-square-difference multiplier" schematic, shown e.g. in Figure 12 of the Final Report of Contract AF19(604)-4535 (P&B Project P-227). The only non-linear elements appearing there are the square-law diodes which may be treated analogous to the cuber in the preceding example.

This proposed generalization of the concept of α is of course not unique, but it should at least encourage the analysis of more specific cases.

b) Effective Antenna Pattern

We may start with the angle-dependent power $f(\theta, \phi)$ as given by the (square-law) detected output of the entire system, and consider this function an effective polar power pattern of the non-linear system. For our experimental arrangement we only determine variation in azimuth and hence $f = f(\phi)$.

c) Equivalent (Electrically) Linear Antenna

By reversing the usual procedure, we may take the effective power pattern of b) above and determine (usually not uniquely) an (electrically) linear antenna system which would produce an equivalent (proportional) pattern. For example, in the case of one of the non-linear systems mentioned in this report we noted that at least for S/N ratios greater than one, the $\frac{\sin 6x}{6 \sin x}$ pattern was equivalent to that of a uniformly illuminated straight-line array of six point sources, uniformly spaced at distance d , where $x = \frac{\pi d}{\lambda} \sin \theta$ (λ = wavelength, θ = angle from normal to array). Note, however, that the reverse procedure of finding an electrically linear antenna system from a given pattern is far from unique (as we have said above), since both aperture geometry as well as amplitude (and phase) distribution over the aperture determine the radiation pattern. We may include the case of the continuous aperture here as the limiting "infinitely dense" version of the discrete point-source array. (In all cases we have of course considered far-field patterns only, for greater simplicity.)

These considerations also presented an interesting problem: Since non-linear analog circuitry has almost unlimited flexibility in the

choice of functions to be generated, would it not be profitable to produce desirable antenna patterns (e.g. of low sidelobe level, high resolution, non-ambiguity, etc.) for which no equivalent linear antenna system appears to exist. For example, the pattern function $(\frac{\sin x}{x})^n$, where n is a large number, offers many desirable receiving characteristics (at least for a single point source transmitter). There is as far as we know no aperture geometry, or amplitude (and phase) distribution which would give a linear antenna system this response.* By combining an algebraic power generator with a continuous, straight, uniformly illuminated aperture, this effective receiving pattern could easily be produced. This example merely serves to illustrate the many potential applications which the non-linear signal processing method showed to possess, some of which were implemented as part of the experimental portion of this contract.

* An exception may be the straight line, uniformly-spaced point source array with a "gabled" (i. e. bilinear) illumination function, which in the limit of vanishing element spacing seems to approach the $(\frac{\sin y}{y})^n$ characteristic. However, even then resolution is sacrificed for the sake of low side lobe level. This arises from the fact that the $x = \pi d/\lambda \sin \theta$ of the uniformly illuminated array becomes $y = \pi d'/\lambda \sin \theta$ for the "gabled" array. Since $d' < d$, the effective aperture is reduced and beamwidth increased, compared to the $(\frac{\sin x}{x})^n$ pattern obtained by means of the analog signal processor (see also S. Silver, M. I. T. Rad. Lab. Series Vol. 12, p. 269).

VI. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

a) Conclusions

Item 1: The successful design and performance of the polynomial generator has shown the practicability of general algebraic non-linear processing of antenna signals. In conjunction with the multipliers we have built, this device, or more extended versions of it, should be adaptable for the simulation of patterns of almost any shape, using a simple interferometer input signal. Noise and circuit component instability will tend to limit the allowable circuit complexity, but the well advanced art of analog computation, together with new solid-state low-noise amplifiers and non-linear devices makes the outlook here more hopeful.

Items 2 and 4: The work here has demonstrated that in the field of radio wave propagation, the multi-frequency method of pattern correlation is feasible and leads to the expected results. Though only the simple case of two frequencies was tested, we believe that it not only points the way to more complex multi-frequency systems but that it showed the workability of an AF, post-detection correlation system, as opposed to the RF and IF correlation systems implemented in other parts of this and preceding contracts. The non-uniform aperture illumination introduced as a requirement for the design of the double-frequency system did not permit optimization, because it was the only one tried, but it should serve as one data point for any future studies of pattern optimization by aperture illumination control.

Item 3: The conclusions in this theoretical study must remain largely tentative until sufficient parallel experimental work has been done. A simple generalized relation of antenna gain, directivity and system efficiency has been proposed as well as a new generalized set of definitions for these and other basic parameters of an antenna system. From the obvious conclusion that the principle of reciprocity must be abandoned for non-linearly processed antenna signals we suggest that the new set of definitions and formulas should be applicable to both transmitting and receiving systems, just like the old set applied to both, because of reciprocity. Then if non-linearity in transmission is encountered, such as propagation through a medium of non-linear properties, the new definitions and formulas will be immediately applicable. No extended analysis has yet been carried on for the presence of noise and/or multiple transmitting sources, and this should perhaps be the next generalization of antenna parameters and formulas in non-linear systems.

b) Recommendations

As we have announced in some of the more recent Quarterly Status Reports, a proposal for future work along lines similar to those carried on under this contract has been submitted to AFCRL during August. The present section of this report therefore freely draws on the ideas expressed in that proposal, but is not limited to them.

The present contract (as well as the preceding one (AF19(604)-4535,) P&B Project P-227) has been concerned with the application of various non-linear operations to signals received on different antenna configurations, both with simulated, continuous as well as discrete "point source" apertures. The basic theoretical background was given in Berman and Clay's paper*, but only a fraction of the many possible circuit arrangements and mathematical operations could be tried so far. Among the operations we attempted were the forming of simple polynomials (up to 5th power) and cross-multiplication of various antenna outputs (see Final Report on Project P-227 and Quarterly Reports of P-264). Rather than exhaust all the different possible combinations of the simple operations tried so far, we believe that other operations and types of transmitting apertures should be investigated in future work. Let us discuss the transmitting apertures first.

* A. Berman and C. S. Clay, Jr. Acoust. Soc. Am. 29, No. 7, 805 (July 1957)

1. Incoherent Extended Transmitting Apertures for Non-Linear Receiving Antenna Systems

As already mentioned in the 5th Quarterly Report on Project P-264, submitted on 9 June 1961, we have been in the process of implementing a simple experiment, not a part of the contractual work, designed to demonstrate the effect of a non-coherent extended (monochromatic) transmitting aperture on a non-linear receiving antenna system. We are simulating this condition by placing two independently driven S-Band transmitting horns equidistant from the receiving antenna, at an angular separation such that both are "visible" on one major lobe of the receiving antenna. This represents a two-point source, non-coherent, extended transmitting aperture, which can be taken as a simple limiting case of the continuous aperture. If this experiment gives significant results, a logical extension in future work would be the use of many discrete, non-coherent point sources, arranged perhaps in a uniformly spaced linear array, for a simple case. For element spacing d this would be the non-coherent analog to the $\frac{\sin N x}{N \sin x}$ pattern aperture, where

$$x = \frac{\pi d}{\lambda} \sin \theta \quad (\theta = \text{angle of incidence})$$

If, $d \ll \lambda$, we would get a fairly close simulation of a non-coherent continuous aperture, the analog to the continuous coherent aperture of pattern $\frac{\sin x}{x}$ for uniform illumination. This example appears rather important, since, unlike the optical case, there are comparatively few naturally occurring incoherent extended sources of RF radiation, such

as perhaps the radio sun, or radio energy scattered by the sea, the fluctuating ionosphere and troposphere, etc. Some of these sources could perhaps be simulated on a small scale in the laboratory or on the antenna range. Possibilities along this line which come to mind include two major groups of devices by which incoherence could be simulated:

- 1) Incoherent reflection or scattering of RF energy.
- 2) Incoherent transmission of RF energy.

The "incoherence" here is, of course, produced by the random mechanical movement of small particles or molecules of the scattering or transmitting medium which is passed on to the radiation. For 1) we could perhaps scatter RF energy off moving water, mercury, or other conducting liquid, suitable vapors or gases confined to a chamber, or finely divided or powdered solids, such as aluminum foil or powder, agitated in an air stream in a closed chamber with RF-transparent windows.

For 2) one could use similar liquids, gases, vapors, powders, etc. consisting of or containing RF-absorbing matter, e.g., graphite powder, graphite suspended in a liquid, or some absorbing gas or vapor, and agitated either artificially or by their inherent Brownian motion, provided the particles are small and light enough.

One difficulty is immediately apparent in all of these schemes. This is the need for collimating the scattered radiation in some fashion so as to concentrate a sizeable portion of the power on the receiving

antenna. No ready-made solution to this problem seems to be available, but the condition of collimation could be approached in a crude fashion by various methods depending on the wave length used. We will leave the details of suggested methods until later, but an obvious method could consist in using microwave lenses and aperture "stops."

2. Non-Linear Operations Other Than Multiplication and Power Raising

In our present work we have made use of both multiplication as well as power raising in processing the antenna signals for optimum antenna system characteristics. A large number of distinct combinations of these two types of operations can be used, in principle at least, to achieve an equally large number of processed "antenna patterns." It is obvious then that the remaining arithmetic operation - division, and its extension to root-taking - ought to be investigated for their usefulness.

Let us first discuss the new effects we could achieve by using these operations. We see immediately that if one divides by a voltage which approaches zero for some values of the incidence angle the resultant "pattern" will exhibit single cusp-shaped "lobes" of high intensity. For instance, if we were to generate the functions $\sec x$, $\csc x$, $\tan x$ or $\cot x$ by the appropriate divisions of the easily obtained basic patterns $\sin x$ and $\cos x$ [e.g. the (phase-shifted) outputs of a two-element additive interferometer] we would, in theory at least, get "infinitely high" cusp-like lobes spaced $\Delta x = \pi$. To get a single cusp (no ambiguity) we could, for example, divide the $\sin x$ pattern

from an out-phased additive two-element interferometer of spacing d into the $\frac{\sin x}{x}$ pattern from a continuous aperture of length d .

This gives $\frac{\sin x}{x} \cdot \frac{1}{\sin x} = \frac{1}{x}$, which rises about as steeply near $x \rightarrow 0$ as the trigonometric ratios above, and is non-ambiguous. Another way, going beyond simple division would consist of performing the analog operation \cos^{-1} on the output signal $\cos x$ of a simple two-element interferometer. The resultant (principal value) quantity x could then be further operated on in many ways to produce a non-ambiguous cusp type pattern, e. g., by division into a constant giving $\frac{a}{x^n}$, $\frac{a}{b x}$ etc. with or without prior power raising or exponentiation.

A few words concerning dividing devices may be in order here. There are several commercial analog dividers on the market. We may mention the G. A. Philbrick Res. model K5-M Universal Multiplier/Divider. Another example is D. H. Schaefer's (NRL) - Static Analogue Divider, described in U. S. Patent 2,948,473, which uses magnetic saturation states and transistorized switching. This inventor remarks in the introductory paragraphs of his patent that many analog dividers using vacuum tube circuits and requiring much greater bulk than his invention are currently available. As a final example, E. R. Wigan¹⁸ in his paper "Measurement of Complex Voltage Ratio" describes an ac potentiometer which would seem to be suitable at least for point-by-point voltage quotient evaluation. It is apparent then that there is no dearth of voltage dividing devices, especially for dc voltages.

3. Iterated Multiplication

It will be recalled that under the preceding contract (AF19(604)-4535) we did not go beyond a single multiplication of antenna signals, nor did we multiply more than once in the work described under Item 4 of the present contract. Since we now have a workable 30 mc multiplier as well as the AF multiplier used in Item 4, we are anxious to apply these to more general analog operations on antenna signals.

One of those we should try is the "iterated multiplication" method in which, for example, a long continuous aperture is simulated by repeatedly multiplying the outputs of properly spaced horns with that of a short continuous aperture, according to the identity

$$\begin{aligned} \frac{\sin x}{x} \cos x \cos 2x \cos 4x \dots \cos 2^n x &= \\ &= \frac{\sin 2^{n+1} x}{2^{n+1} x} \end{aligned} \quad (10)$$

This operation had been attempted unsuccessfully by us with the 2 mc multipliers, and should now be repeated using the 30 mc multipliers in which the drift problems are much less severe.

In connection with this work it will probably be useful to investigate more fully some of the other analog multiplier devices we have read about in the literature, but did not use. Two devices which come to mind here are the "Quadratron" multiplier and the various Hall effect devices.

4. Generalization to Multiple Sources, Presence of Noise Fields and Non-Uniformly Illuminated Receiving Apertures

Under this proposed effort we propose to extend the analytical work of Item 3 to the considerably more difficult cases of multiple (generally incoherent but close in frequency) transmitting sources, and/or noise sources simultaneously being received by an antenna system whose outputs are processed in some non-linear fashion. We realize that quite a few theoretical papers have appeared in the literature on this very subject. We therefore suggest that we could perform experimental studies required to substantiate some of these results and to carry through our own theoretical analyses.

SECTION VII. PARTICIPATING SCIENTISTS AND ENGINEERS

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In addition to the above references, the bibliography in the Final Report on the preceding contract (AF 19(604)-4535) will be found useful.